Bello Honest 18/ENG04/023

Electrical

a) $\frac{dy}{dx} = 2\sinh x - y\tanh x$

$\frac{dy}{dx} + y\tanh x = 2\sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x = u$

$\int \frac{\sinh x}{u} dx$

$u = \cosh x$ $\quad dx = \frac{du}{\sinh x}$

$du/dx = \sinh x$

$\int \frac{1}{u} du = \ln u = \ln \cosh x$

$I.F = e^{\int P dx} = e^{\ln \cosh x}$

$I.F = \cosh x$

Then $y(I.F) = \int Q.(I.F)\, dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x\, dx$

$2\sin x \cos x = \sin(2x)$

$2\sinh x \cosh x = \sinh(2x)$

$y\cosh x = \int \sinh 2x\, dx$

$y \cdot \cosh x = \frac{1}{2} \cdot x \cosh 2x + C$

$\cosh x\, y = \cosh 2x + C$

$y = \frac{\cosh 2x + C}{2}$

$\frac{\cosh 2x}{\cosh x}$

$y = \frac{\cosh 2x + 2c}{\cosh x}$

Let $2C = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2 \int dx = 2x$

$Q = e^{2x}$

$I.F = e^{\int P dx} = e^{2x}$

$y.(I.F) = \int Q.(I.F)\, dx$

$y \cdot e^{2x} = \int e^{2x} \cdot e^{3x}\, dx$

$y \cdot e^{2x} = \int e^{5x}\, dx$

$y \cdot e^{2x} = \frac{1}{5} e^{5x} + C$

$y = \frac{\frac{1}{5} e^{5x} + C}{e^{2x}}$

c) $\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\therefore \int \frac{dy}{dx} = \int x + 2 - \frac{3}{x}\, dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + C$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$\frac{1}{y^3} y^{-3} + y^{-2} \frac{1}{x} = 1$ ——1

$z = y^{1-n}$, $n = 3$

$z = y^{-2}$ ——2

$\therefore \frac{dz}{dx} = -2y^{-3} \frac{dy}{dx}$ ——3

eqn x (-n)

$-2y^{-3} \frac{dy}{dx} - \frac{2y^{-2}}{x} = -2$

$-2 \cdot \frac{dy}{y} = -2y^{-3} \frac{dy}{dx}$

Sub eq. 2 & 3 into 4

$\frac{dy}{dx} - \frac{2y}{x} = -2$

$P = -2/x, \quad Q = 2$

$\int P dx = -2\ln x$

$If = e^{-2\ln x} = x^{-2}$

2. y IF $= \int Q.IF \cdot dx$

$2x^2 = \int -2x^{-2} dx$

$= \frac{-2x^{-1}}{-1} + c$

$2x^{-2} = 2x^{-1} + c$

$2 = \frac{2x^{-1}}{x^2} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$y^{-2} = x(2 + cx)$

$\frac{1}{y^2} = x(2 + cx)$

$y^2 = \frac{1}{x(2 + cx)}$

$y = \sqrt{\frac{1}{x(2 + cx)}}$

$y = \frac{1}{\sqrt{x(2+cx)}}$

e) $x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$

$\frac{dy}{dx} = x \sin 3x + 4$

$\int \frac{dy}{dx} = \int x^4 \sin 3x + \int 4x^{-2}$

$= \frac{1}{3}\cos 3x + \int \frac{1}{3}\cos 3x + 4x^{-1}$

$y = \frac{\sin 3x}{a} - \frac{x\cos 3x}{3} - \frac{4}{x}$

f) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{2(vx)^3}{x^3 + v^2x^3}$

$v + x\frac{dv}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$

$\frac{x\,dv}{dx} = \frac{2v^3}{1+v^2}$

$= \frac{2v^3 - v - v^3}{1+v^2}$

$x\,dv = v^3$

$\frac{1+v^2}{v^3 - v} dv = \frac{1}{x} dx$

$v(v-1)(v+1) = v^3 - V$

$\frac{1-v^2}{v^3-v}$

$1 + v^2 =$

$A\,c($

$1+$

$2=$

$1+$

$$\frac{1-v^2}{v^3-v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$$

$1+v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$

$A = (v)(v-1)$, $v=1$

$1+1^2 = B(1)(2)$

$2 = B(2)$

$\therefore B = 1$

$v = -1$

$-1^2 = C(-1)(-1-1)$

$2 = 2C$

$\therefore C = 1$

$v = 0$

$1+(0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = A-1$

$\therefore A = -1$

$$\int \left(\frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1}\right) dv = \int dx \frac{1}{x}$$

$$\int \frac{-1}{v} + \int \frac{1}{v-1} + dv + \int \frac{1}{v+1} dv = \frac{1}{x} dx$$

$-\ln v + \ln(v+1) + \ln(v-1) = \ln x + C$

$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$

$v^2 - 1 = Ax$

0

$\left(\frac{y}{x}\right)^2 = v \times v = \frac{y}{x}$

$\frac{y}{x} - 1 = Ax$

$\frac{y}{x}$

$\frac{y^2}{x^2} - 1 = Ax$

$\frac{y^2}{x^2} - 1 = Ay$

$\frac{y^2 - x^2}{x^2} = Ay$

$y^2 - x^2 = Ayx^2$

$y^2 = Ax^2y + x^2$

$y^2 = x^2(Ay + 1)$