OBE CORNELIUS MBA
18/ENG06/049
MATHEMATICS
ENG 284

a) $\frac{dy}{dx} = 2\sinh x - y\tanh x$

$dy/dx + y\tanh x = \sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x = u$

$\int \frac{\sinh x}{4} dx$

$u = \cosh x$ $\qquad dx = \frac{du}{\sinh x}$

$du/dx = \sinh x$

$\int \sinh x/u \cdot du/\sinh x$

$\int \frac{1}{u} du = \ln u = \ln \cosh x$

$IF = e^{\int P dx} = e^{\ln \cosh x}$

Then $y IF = \int Q \cdot IF\, dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x\, dx$

$2\sin x \cos x = \sin(2x)$

$2\sinh x \cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x\, dx$

$y \cdot \cosh x = \frac{1}{2} \cdot 2\cosh 2x + C$

$\cosh x\, y = \cosh 2x + C$

$y = \frac{\frac{\cosh 2x}{2} + c}{\cosh x}$

$y = \frac{\cosh 2x + 2C}{\cosh x}$

Let $2C = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $dy/dx + 2y = e^{3x}$

$P = 2$ $\qquad \int P dx = 2x$

$Q = e^{2x}$

$IF\ e^{\int P dx} = e^{2x}$

$y \cdot IF = \int Q \cdot IF\, dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}\, dx$

$y \cdot e^{2x} = \int e^{5x} dx$

$y \cdot e^{2x} = \frac{1}{5} e^{5x} + c$

$y = \frac{\frac{1}{5} e^{5x} + c}{e^{2x}}$

c) $-dy/dx = x^2 + 2x - 3$

$dy/dx = x + 2 - \frac{3}{x}$

$\int dy/dx = \int x + 2 - \frac{3}{3}\, dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + C$

d) $dy/dx + y/x = y^3$

$dy/dx\, y^{-3} + y^{-2}/x = 1$ — eqn (1)

$Z = y^{1-n}$, $n = 3$

$Z = y^{1-3} = Z = y^{-2}$ — eqn (2)

$dz/dy = -2y^{-3} dy/dx$ — eqn (3)

Then Multiply eqn by $1-n$

$-2y^{-3} \frac{dy}{dx} - \frac{2y^{-2}}{x} = -2$

$\frac{dz}{dy} = \frac{-2y^{-3} dy}{dx}$ — eqn (4)

Sub eqn 2 & 3 into eqn (4)

$\frac{dz}{dy} - \frac{2z}{x} = -2$

$P = -2/x$, $Q = -2$

$\int P dx = -2\ln x$

$IF = e^{-2\ln x} = e^{-7}$

$Z \cdot IF = \int Q \cdot IF \cdot dx$

$Z \cdot x^{-2} = \int -2x^{-2} dx$

$= \frac{-2x^{-1}}{-1} + C$

$2 \cdot x^{-2} = 2x^{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2, \; z = x(2+cx)$

$z = y^{-2}$

$y^{-2} = x(2+cx)$

$1/y^2 = x(2+cx)$

$y^2 = \frac{1}{x(2+cx)}$

$\therefore y = \sqrt{\frac{1}{x(2+cx)}}$

$y = \frac{1}{\sqrt{x(2+cx)}}$

e) $x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$

$dy/dx = x \sin 3x + 4$

$\int dy/dx = \int x^4 \sin 3x + \int 4x^{-2}$

$= 1/3 \cos 3x + \int 1/3 \cos 3x + \frac{4x^{-1}}{-1}$

$= \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1}$

$y = \frac{\sin 3x}{9} - \frac{x \cos 3x}{3} - \frac{4}{x}$

f) $(x^3 + xy^2)\, dy/dx = 2y^3$

$y = vx$

$dy/dx = u + x\, du/dx$

$u + x\, du/dx = \frac{2(ux)^3}{x^3 + u^2x^3}$

$u + x\, du/dx = \frac{x^3(2u^3)}{x^3(1+u^2)}$

$x\frac{du}{dx} = \frac{2u^3}{1+u^2}$

$x\, dv = u^3$

$\frac{1+v^2}{v^3 - v} dv = \frac{1}{x} dx$

$v(v-1)(v+1) = v^3 - v$

$\frac{1+v^2}{v^3-v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$1+v^2 = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$1+u^2 = A(v-1)(v+1) + B(u)(v+1) + C(v)$

$(v-1), v = 1$

$1 + 1^2 = B(1)(2)$

$2 = B(2) \quad \therefore B = 1, \; v = -1$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = A - 1$

$\therefore A = -1$

$\int \left[\frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right] dv = \int dx \frac{1}{x}$

$\int -1/v + \int \frac{1}{v-1} + dv + \int \frac{1}{v+1} dv = \int \frac{1}{x} dv$

$-\ln u + \ln(v-1) + \ln(v+1) = \ln x + C$

$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$

$\frac{v^2 - 1}{u} = Ax, \quad y = vx$

$\frac{\left(\frac{y}{x}\right)^2 - 1}{(y/x)} = Ax$

$\frac{y^2}{x^2} - 1 = A\cancel{x} \cdot \frac{y}{\cancel{x}}$

$\frac{y^2}{x^2} - 1 = Ay$

$\frac{y^2 - x^2}{x^2} = Ay$

$y^2 - x^2 = Ayx^2$

$y^2 = Ax^2y + x^2$

$y^2 = x^2(Ay + 1)$