Substitute for $\frac{dz}{dx}$

$$\frac{dz}{dx} - \frac{2z}{x} = -2$$

Let $\frac{-2}{x} = M$

$-2 = N$

$IF = e^{\int M dx}$

$\therefore \int M dx = -2\ln x$

$IF = e^{-2\ln x} = x^{-2}$

$y \cdot IF = \int N \cdot IF\, dx$

$y \cdot x^{-2} = \int -2 \cdot x^{-2}\, dx$

$y x^{-2} = 2x + c$

$y = 2x^3 + x^2 c$

5) $x^4 \frac{dy}{dx} = x^3 \sin 3x + 4$

$$\frac{dy}{dx} = \sin 3x + \frac{4}{x^3}$$

$y = \int \sin 3x + \frac{4}{x^3}$

$$y = \frac{-\cos 3x}{3} - \frac{4}{2x^2}$$

$$y = \frac{-\cos 3x}{3} - \frac{2}{x^2}$$

6 $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

Recall, $\frac{dy}{dx} = V + x\frac{dv}{dx}$, and $y = Vx$

$x^3(1+v^2)\frac{dy}{dx} = 2V^3x^3$

$(1+v^2)\frac{dy}{dx} = 2V^3$

$(1+v^2)\left(V + x\frac{dV}{dx}\right) = 2V^3$

$$\frac{V(v^2-1)}{1+v^2} = x\frac{dv}{dx}$$

$$\frac{dx}{x} = \frac{1+v^2}{V(v^2-1)} \cdot dv$$

Resolving RHS into partial fractions

$$\int \frac{1}{x} dx = \int \left(\frac{-1}{v} + \frac{1}{V-1} + \frac{1}{V+1}\right) dv$$

$\ln x = -\ln v + \ln(V-1) + \ln(V+1) + c$

$$\ln x = \ln \frac{(V-1)(V+1)}{V} + c$$

$y = Vx$

$V = \frac{y}{x}$

$$\ln x = \ln \left[\frac{\left(\frac{y}{x}-1\right)\left(\frac{y}{x}+1\right)}{\frac{y}{x}}\right] + c$$

$$\ln x = \ln \frac{\left(\frac{y^2}{x^2} - 1\right)}{\frac{y}{x}} + c$$

$$\ln x = \ln \left(\frac{y^2 - x^2}{x^2}\right) + c$$

$$\ln x = \ln \left(\frac{y^2 - x^2}{x^2}\right) + \ln A$$

$$\ln x - \ln \left(\frac{y^2 - x^2}{x^2}\right) = \ln A$$

$$\ln \frac{x(x^2)}{y^2 - x^2} = \ln A$$

$$\frac{x^3}{y^2 - x^2} = A$$

$A(y^2 - x^2) = x^3$

$$y^2 - x^2 = \frac{x^3}{A}$$

$$y^2 = \frac{x^3}{A}$$

$$y^2 = \frac{x^3}{A} + x^2$$

$$y = \sqrt{\frac{x^3 + Ax^2}{A}}$$

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$\frac{dy}{dx} + y\tanh x = 2\sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$I_F = e^{\int P dx}$

$\int P dx = \int \tanh x \, dx$

$= \int \frac{\sinh x}{\cosh x} dx$

$= \int \frac{\sinh x}{h} dx$

$x = \cosh x$

$\frac{dy}{dx} = \sinh x$

$dx = \frac{dy}{\sinh x}$

$= \int \frac{1}{h} dx$

$= \ln h$

$= \ln(\cosh x)$

$I_F = e^{\ln\cosh x} = \cosh x$

$y I = \int Q \cdot I_F dx$

$y\cosh x = \int 2\sinh x \cdot \cosh x \, dx$

$y\cosh x = 2\int \sinh x \cdot \cosh x \, dx$

Let $\cosh x = u$

$\frac{du}{dx} = \sinh x$

$dx = \frac{du}{\sinh x}$

$y\cosh x = 2\int \sinh x \cdot u \cdot \frac{du}{\sinh x}$

$y\cosh x = 2\int u \, du$

$y\cosh = u^2$

$y\cosh x = \cosh^2 x + c$

$y = \cosh x + \frac{c}{\cosh x}$

2) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2$

$Q = e^{3x}$

$I_F = e^{\int P dx}$

$\int P dx = \int 2 dx$

$= 2x$

$I_F = e^{2x}$

$y \cdot I_F = \int Q \cdot I_F dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} dx$

$y \cdot e^{2x} = \int e^{5x} dx$

$ye^{2x} = \frac{e^{5x}}{5} + c$

$y = \frac{e^{3x}}{5} + \frac{c}{e^{2x}}$

3) $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

$y = x^2 + 4x - 6\ln x + 2c$

4) $\frac{dy}{dx} + \frac{y}{x} = y^4$

$y^{-3}\frac{dy}{dx} + \frac{y^{-2}}{x} = 1$

Let $y^{-2} = z$

$\frac{1}{x} = P$

$1 = Q$

$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

$= -2y^{-3} \cdot \frac{dy}{dx}$

Multiply through by -2

$-2y^{-3}\frac{dy}{dx} - \frac{2y^{-2}}{x} = -2$