

$$-2y^{-3} \frac{dy}{dx} - \frac{2y^2}{2} = -2$$

Sub for dz/dx

$$\frac{dz}{dx} - \frac{2}{x} z = -2$$

let $\frac{-2}{x} = M$ $-2 = N$

IF = $e^{\int M dx}$

$$\int M dx = 2 \ln x$$

$$IF = e^{5-2 \ln x} = x^{-2}$$

$$y \cdot IF = \int N \cdot IF dx$$

$$y \cdot x^{-2} = \int -2 \cdot x^{-2} dx$$

$$y \cdot x^{-2} = 2x + C$$

$$y = 2x^3 + x^2 C$$

c) $x^3 \frac{dy}{dx} = x^3 \sin 3x + IF$

$$\frac{dy}{dx} = \sin 3x + \frac{1}{x^3}$$

$$y = \int \sin 3x + \frac{1}{x^3}$$

$$y = -\frac{\cos 3x}{3} - \frac{2}{x^3}$$

$$y = \frac{\sin 3x}{3} - \frac{\cos 3x}{3} - \frac{2}{x^3}$$

b) $(x^3 + xy^2) \frac{dy}{dx} = 2y^3$

recall, $\frac{dy}{dx} = v + x \frac{dv}{dx}$, and $y = vx$

$$x^3(1+v^2) \frac{dy}{dx} = 2v^3 x^3$$

$$(1+v^2) \frac{dy}{dx} = 2v^3$$

$$(1+v^2) \left(v + \frac{dv}{dx} \right) = 2v^3$$

$$\frac{v(v^2-1)}{1+v^2} = \frac{1}{x} \frac{dv}{dx}$$

$$\frac{dx}{x} = \frac{1+v^2}{v(v^2-1)} \cdot dv$$

Resolving RHS into partial fraction

$$\int \frac{1}{x} dx = \int \left(\frac{1}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right) dv$$

$$\ln x = -\ln v + \ln(v-1) + \ln(v+1) + C$$

$$\ln x = \frac{\ln(v-1)(v+1)}{v} + C$$

$$y = vx$$

$$v = y/x$$

$$\ln x = \ln \left[\frac{(y/x - 1)(y/x + 1)}{y/x} \right] + C$$

$$\ln x = \ln \left(\frac{y^2 - x^2}{x^2} \right) + C$$

$$\ln x = \ln \left(\frac{y^2 - x^2}{x^2} \right) = \ln A$$

$$\frac{\ln x(x^2)}{y^2 - x^2} = \ln A$$

$$\frac{x^3}{y^2 - x^2} = A$$

$$A(y^2 - x^2) = x^3$$

$$y^2 - x^2 = \frac{x^3}{A}$$

$$y^2 = \frac{x^3}{A} + x^2$$

$$y = \sqrt{\frac{x^3 + Ax^2}{A}}$$

1) $\frac{dy}{dx} + y \tanh x = 2 \sinh 2x$

$P = \tanh x$ $Q = 2 \sinh 2x$

IF = $e^{\int P dx}$

$\int P dx = \int \tanh x dx$

$= \int \frac{\sinh x}{\cosh x} dx$

$= \int \frac{\sinh x}{x} dx$

$u = \cosh x$

$\frac{du}{dx} = \sinh x$

$dx = \frac{du}{\sinh x}$

$\frac{1}{\sinh x}$

$= \int \frac{1}{u} du$

$= \ln u$

$= \ln(\cosh x)$

IF = $e^{\ln \cosh x} = \cosh x$

$y \cdot I = \int Q \cdot IF dx$

$y \cosh x = \int 2 \sinh x \cdot \cosh x dx$

$y \cosh x = 2 \int \sinh x \cdot \cosh x dx$

let $\cosh x = u$

$\frac{du}{dx} = \sinh x$

$dx = \frac{du}{\sinh x}$

$y \cosh x = 2 \int \sinh x \cdot u \cdot \frac{du}{\sinh x}$

$y \cosh x = 2 \int u du$

$y \cosh x = u^2$

$y \cosh x = \cosh^2 x + C$

$y = \cosh x + \frac{C}{\cosh x}$

2) $\frac{dy}{dx} + 2y = e^{2x}$

$P = 2$ $Q = e^{2x}$

IF = $e^{\int P dx}$

$\int P dx = \int 2 dx$

$= 2x$

IF = e^{2x}

$y \cdot IF = \int Q \cdot IF dx$

$y \cdot e^{2x} = \int e^{2x} \cdot e^{2x} dx$

$y \cdot e^{2x} = \int e^{4x} dx$

$y \cdot e^{2x} = \frac{e^{4x}}{4} + C$

$y = \frac{e^{2x}}{4} + \frac{C}{e^{2x}}$

3) $x \frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$y = \frac{x^2}{2} + 2x - 3 \ln x + C$

$y = x^2 + 4x - 6 \ln x + 2C$

4) $\frac{dy}{dx} + \frac{y}{x} = y'$

$y^{-2} \frac{dy}{dx} + \frac{y^{-2}}{x} = 1$

let $y^{-2} = z$

$\frac{1}{x} = P$ $1 = Q$

$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

$= -2y^{-2} \cdot \frac{dy}{dx}$

multiply through by -2