Mechatronics Engineering

(2) $dy/dx = 2\sinh x - y\tanh x$

(1) $dy/dx + y\tanh x = 2\sinh x$

$P = \tanh x \quad , Q = 2\sinh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} \cdot dx$

$= \ln\cosh x$

$I.F = e^{\int P dx} = e^{\ln\cosh x} = \cosh x$

$y \cdot IF = \int Q \cdot IF \cdot dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x \, dx$

$2\sinh x \cdot \cosh x = \sinh 2x$

$y \cdot \cosh x = \int \sinh 2x \cdot dx$

$y \cdot \cosh x = 1/2 \times 2\cos 2x + C$

$y \cdot \cosh x = \cosh 2x + C$

$y = \frac{\cosh 2x + C}{\cosh x}$

(b) $dy/dx + 2y = e^{3x}$

$P = 2 \, , Q = e^{3x}$

$\int P dx = 2x \, , I.F = e^{2x}$

$y \cdot IF = \int Q \cdot IF \, dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} dx$

$y e^{2x} = \int e^{5x} dx$

$y e^{2x} = \frac{e^{5x}}{5} + C$

$y = \frac{e^{3x}}{5} + Ce^{-2x}$

(c) $x \, dy/dx = x^2 + 2x - 3$

$dy = x + 2 - \frac{3}{x} dx$

$\int dy = \int x + 2 - \frac{3}{x} dx$

$y = \frac{x^2}{2} + 2x - \ln x + C$

(d) $x^2 \, dy/dx = x^3 \sin 3x + 4$

$dy/dx = x\sin 3x + 4x^{-2} dx$

$\int dy = \int x\sin 3x + 4x^{-2} dx$

$y = \frac{1}{3} \cdot \cos 3x + \int \frac{1}{3} \cdot \cos 2x + \left(\frac{4x^{-1}}{-1}\right)$

$y = \frac{-x\cos 3x}{3} - \left(\frac{-\sin 3x}{4}\right) - 4x^{-1}$

$y = \frac{\sin 3x}{4} - \frac{x\cos 3x}{3} - \frac{4}{3}$

(e) $dy/dx + y/x = y^3$

$dy/dx + \left(\frac{1}{x}\right)y = y^3$

$z = y^{1-3} = y^{-2}, \quad dz/dx = -2 \cdot y^{-3} dy/dx$

$y^{-3} dy/dx + \left(\frac{1}{x}\right) y^{-2} = 1$

$-2y^{-3} dy/dx - \left(\frac{2}{x}\right) y^{-2} = -2$

$dz/dx - \frac{2}{x} z = -2$

$I.F = x^{-2}$

$z \cdot IF = \int Q \cdot IF dx$

$\frac{z}{x^2} = -\int \frac{2}{x^2} dx + C$

$\frac{z}{x^2} = \frac{2}{x} + C$

$\frac{z}{x^2} = \frac{2 + xC}{x}$

$z = 2x + x^2 C$

$y^2 = \frac{1}{2x + x^2 C}$

(f) $(x^3 + xy^2) dy/dx = 2y^3$

$dy/dx = \frac{2y^3}{x^3 + xy^2}$

$V + x \, dv/dx = \frac{2V^3 x^3}{x^3 + V^2 x^3}$

$V + x \, dy/dx = \cancel{2V^3} \; 2V \; \frac{2V^3}{1 + V^2}$

$$x\frac{dv}{dx} = \frac{2V^3}{1+V^2} - V$$

$$x\frac{dv}{dx} = \frac{2V^3 - (V(1+V^2))}{1+V^2}$$

$$x\,dv/dx = \frac{2V^3 - V - V^3}{1+V^2}$$

$$x \cdot dv = V^3$$

$$\frac{1+V^2}{V^3-V} \cdot dv = \frac{1}{x} \cdot dx$$

$$V(V-1)(V+1) = V^3 - V$$

$$\frac{1+V^2}{V^3-V} = \frac{A}{V} + \frac{B}{V-1} + \frac{C}{V+1}$$

$$1+V^2 = A(V-1)(V+1) + B(V)(V+1) + C(V)(V-1)$$

$$1+1^2 = B(1)(2)$$

$$2 = 2B$$

$$B = 1$$

$$V = -1$$

$$1+(-1)^2 = C(-1)(-1-1)$$

$$2 = 2C$$

$$C = 1$$

$$V = 0$$

$$1+(0)^2 = A(0-1)(0+1)$$

$$1 = A(-1)(1)$$

$$1 = -A$$

$$A = -1$$

$$\int\left(-\frac{1}{V} + \frac{1}{V-1} + \frac{1}{V+1}\right)dv = \int \frac{1}{x}\,dx$$

$$-\ln V + \ln(V-1) + \ln(V+1) = \ln x + \ln A$$

$$\therefore \frac{V^2-1}{V} = Ax$$

$$\frac{y^2}{x^2} - 1 = Ax - y/x$$

$$\frac{y^2}{x^2} - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$y^2 - x^2 = Ay\,x^2$$

$$y^2 = Ax^2y + x^2$$

$$y^2 = x^2(Ay + 1)$$