EMETH GALEB 18-03-20
19/ENG03/033
CIVIL ENGINEERING.
ENG 284 (MATHS)

a) $dy/dx = 2\sinh x - y\tanh x$

$dy/dx + y\tanh x = 2\sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int p\,dx = \int \tanh x = \int \frac{\sinh x}{\cosh x}dx$

$\cosh x = u$

$\int \frac{\sinh x}{u}dx$

$u = \cosh x \quad dx = du/\sinh x$

B $du/dx = \sinh x$

$\int \sinh x/u \cdot du/\sinh x$

$\int 1/u\,du = \ln u = \ln\cosh x$

$IF = e^{\int P dx} = e^{\ln\cosh x}$

$IF = \cosh x$

Then $y\,IF = \int Q \cdot IF\,dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x\,dx$

$2\sin x\cos x = \sin(2x)$

$2\sinh x\cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x\,dx.$

$y \cdot \cosh x = 1/2 \cdot \cosh 2x + c$

$\cosh x\,y = \cosh 2x + c$

$y = \frac{\cosh 2x/2 + c}{\cosh x}$

$y = \frac{\cosh 2x + 2c}{\cosh x}$

Let $2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $dy/dx + 2y = e^{3x}$

$P = 2 \quad \int P dx = 2x$

$Q = e^{3x}$

$IF = e^{\int P dx} = e^{2x}$

$y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}dx$

$y \cdot e^{2x} = \int e^{5x}dx$

$y \cdot e^{2x} = 1/5 e^{5x} + c$

$y = \frac{1/5 e^{5x} + c}{e^{3x}}$

c) $x\,dy/dx = x^2 + 2x - 3$

$dy/dx = x + 2 - 3/x$

$\int dy/dx = \int x + 2 - 3/x\,dx$

$y = x^2/2 + 2x - 3\ln x + c$

d) $dy/dx + y/x = y^3$

$dy/dx\,y^{-3} + y^{-2}/x = 1$ — (i)

$z = y^{1-n}, \quad n = 3$

$z = y^{1-3}, \quad z = y^{-2}$ — (ii)

$dz/dy = -2y^{-3}\,dy/dx$ — (iii)

Then multiply eq (i) by (1-n)

$-2y^{-3}dy/dx - 2y^{-2}/x = -2$

and $dz/dy = -2y^{-3}\,dy/dx$

Sub eq (ii) & (iii) into (d)

$dz/dy - 2z/x = -2$

$P = -2/x, \; Q = -2$

$\int P dx = -2\ln x$

$IF = e^{-2\ln x} = x^{-2}$

$z \cdot If = \int Q \cdot If \cdot dx$

$z \cdot x^{-2} = \int -2x^{-2}\, dx$

$= \frac{-2x^{-1}}{-1} + c$

$2x^{-2} = 2x^{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$y^{-2} = x(2 + cx)$

$1/y^2 = x(2 + cx)$

$y^2 = 1/x\,(2 + cx)$

$\therefore y = \sqrt{1/x(2 + cx)}$

$y = 1/\sqrt{x(2 + cx)}$

e) $x^2\, dy/dx = x^3 \sin 3x + 4$

$dy/dx = x \sin 3x + 4$

$\int dy/dx = \int x^4 \sin 3x + \int 4x^{-2}$

$= 1/3 \cos 3x + \int 1/3 \cos 3x + 4x^{-1}$

$= -x \cos 3x/3 + \sin 3x/4 - 4x$

$y = \sin 3x/4 - x \cos 3x/3 - 4/x$

f) $(x^3 + xy^2)\, dy/dx = 2y^3$

$y = dx$

$dy/dx = v + x\, dv/dx$

$v + x\, dv/dx = 2(vx)^3/x^3 + v^2x^3$

$v + x\, dv/dx = \cancel{3v^3}\,(2v^3)/x^3(1+v^2)$

$x\, dv/dx = 2v^3/1 + v^2$

$\frac{-2v^3 - v(1+v^2)}{1+v^2}$

$= \frac{2v^3 - v - v^3}{1+v^2}$

$x\, dv = v^3$

$\frac{1+v^2}{v^3 - v}\, dv + 1/x\, dx$

$v(v-1)(v+1) = v^3 - v$

$\frac{1+v^2}{v^3 - v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$1 + v^2 = A(v-1)(v+1) + B(v)(v+1)$

$+ C(v)(v-1),\ v = 1$

$1 + 1^2 = B(1)(2)$

$2 = B(2)$

$\therefore B = 1$

$v = -1$

$1 + (-1)^2 = C(-1)(-1-1)$

$\cancel{2} = \cancel{2}C$

$\therefore C = 1$

$x = 0$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = A - 1$

$\therefore A = -1$

$\int \left[-\frac{1}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right] dv = \int dx\, 1/x$

$\int -1/v\, dv + \int 1/v-1 + dv + \int 1/v+1\, dv =$

$\int 1/x\, dv$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + c$

$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$

$\frac{v^2 - 1}{v} = Ax$

$y = vx \therefore v = y/x$

$$\frac{(y/x)^2 - 1}{(y/x)} = Ax$$

$$y^2/x^2 - 1 = Ax \cdot y/x$$

$$y^2/x^2 - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$y^2 - x^2 = Ayx^2$$

$$y^2 = Ax^2y + x^2$$

$$y^2 = x^2(Ay + 1).$$