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ENG282

ⓐ $\frac{dy}{dx} = 2\sinh x - y\tanh x$

$\frac{dy}{dx} + y\tanh x = \sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P\,dx \int \tanh x = \int \frac{\sinh x}{\cosh}\,dx$

$\cosh x = u$

$\int \frac{\sinh x}{\cancel{4}}\,dx$

$u = \cosh x \qquad dx = \frac{du}{\sinh x}$

$\frac{du}{dx} = \sinh x$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x}$

$\int \frac{1}{u}\,du = \ln u = \ln\cosh x$

$IF = e^{\int P\,dx} = e^{\ln\cosh x}$

$IF = \int Q \cdot IF\,dx$

$IF = \int Q \cdot IF\,dx$

Then $y\,IF = \int Q \cdot IF\,dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cos x\,dx$

$2\sin x\cos x = \sin(2x)$

$2\sinh x\cosh x = \sinh(2x)$

$y \cdot \cosh x = \frac{1}{\cancel{2}} \cdot \cancel{2}\cosh 2x + c$

$\cosh x y = \cosh 2x + c$

$y = \frac{\frac{\cosh 2x + c}{2}}{\cosh x}$

$y = \frac{\cosh 2x + AC}{\cosh x}$

Let $2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$ //

b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2 \quad \int P\,dx = 2x$

$Q = e^{3x}$

$IF = e^{\int P\,dx} = e^{2x}$

$\therefore y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}\,dx$

$y \cdot e^{2x} = \int e^{5x}\,dx$

$y \cdot e^{2x} = \frac{1}{5}e^{5x} + C$

$y = \frac{\frac{1}{5}e^{5x} + c}{e^{3x}}$ //

c) $2\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\therefore \int \frac{dy}{dx} = \int \frac{x + 2 - 3}{3}\,dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + C$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$\frac{dy}{dx}\,y^{-3} + \frac{y^{-2}}{x} = 1$ ... eq ①

$z = y^{1-n} \qquad n = 3$

$z = y^{1-3},\ z = y^{-2}$ ... ②

$\therefore \frac{dz}{dy} = -2y^{-3}\, dy/_{dx}$ — ③

then multiply eqn ① by $1-n$

$-2y^{-3} dy/_{dx} \; \frac{-2y^{-2}}{x} = -2$

and $\frac{dz}{dy} = \frac{-2y^{-3}dy}{dx}$

Sub eq 2 & 3 into 4

$\frac{dz}{dy} - \frac{2z}{dx} = -2$

i.e. $P = -2/_x$, $Q = -2$

$\int P dx = -2\ln x$

$If = e^{-2\ln x} = x^{-2}$

$z \cdot If = \int Q \cdot IF dx$

$z \cdot x^{-2} = \int -2x^{-2} dx$

$z = \frac{-2x^{-1}}{-1} + C$

$zx^{-2} = 2x^{-1} + C$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{C}{x^{-2}}$

$z = 2x + Cx^2$

$z = x(2 + Cx)$

$z = y^{-2}$

$y^{-2} = x(2 + Cx)$

$1/y^2 = x(2 + Cx)$

$y^2 = \frac{1}{x(2+Cx)}$

$\therefore y = \sqrt{\frac{1}{x(2+Cx)}}$

$y = \frac{1}{\sqrt{x}(2+Cx)}$

⑥ $x^2\, dy/_{dx} = x^2 \sin 3x + 4$

$dy/_{dx} = x \sin 3x + 4$

$\int dy/_{dx} = \int x^4 \sin 3x + \int 4x^{-2}$

$= 1/3 \cos 3x + \int 1/3 \cos 3x + \frac{4x^{-1}}{-1}$

$= \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} - \frac{4x^{-1}}{-1}$

$= \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1}$

$y = \frac{\sin 3x}{9} - \frac{x\cos 3x}{3} - \frac{4}{x}$

⑧ $(x^3 + xy^2)\, dy/dx = 2y^3$

$y = vx$

$dy/_{dx} = v + x\, du/dx$

$v + x\, du/dx = 2(v_2)^3$

$v + x\, du/dx = \frac{x^3(2x^3)}{x^3(1+v^2)}$

$= \frac{2v^3 - v(1+v^2)}{1+v^2}$

$= \frac{2v^3 - v - v^3}{1+v^2}$

$\frac{x\,du}{1+v^2} = v^3$

$\frac{1+v^2}{v^3 - v} = du = 1/_x\, dx$

$v(v-1)(v+1) = v^3 - v$

$$\frac{1+v^2}{v^3-v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$$

$1+v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$, $v=1$

$1+1^2 = B(1)(2)$

$2 = B(2)$

$\therefore B = 1$

$v = -1$

$1+(-1)^2 = C(-1)(-1-1)$

$2 = x$

$\therefore C = 1$

$v = 0$

$1+(0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$2 = A - 1$

$\therefore A = -1$

$$\int\left[\frac{-1}{v} + \frac{1}{v+1} + \frac{1}{v+1}\right]du = \int \frac{dx}{x}$$

$$\int \frac{-1}{v}du + \int \frac{1}{v-1}du + \int \frac{1}{v+1}du$$

$$= \int \frac{1}{x}du$$

$-\ln v + \ln(v-1) + \ln(v+1) = \theta$

$= \ln(v-1)(v+1) - \ln v = \ln x$

$$= \frac{v^2-1}{v} = Ax$$

$y = vx \quad \therefore x = y/x$

$$\left(\frac{y}{x}\right)^2 - 1 = Ax$$

$$\frac{y^2}{x^2} - 1 = Ax - \frac{y}{\cancel{x}}$$

$$\frac{y^2}{x^2} - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$\frac{y^2 - x^2}{\cancel{x^2}} \cdot y^2 - x^2 = Ayx^2$$

$y^2 = Ax^2y + x^2$

$y^2 = x^2(Ay+1)$ //