Olaschde Olaitan oghenerairna

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Mechatronics Engineering

ENG 282 Assignment II

a) $\frac{dy}{dx} - 2\sinh x - y\tanh x$

$\frac{dy}{dx} + y\tanh x = 2\sinh x$

$P = \tanh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x = u \quad \frac{dy}{dx} = \sinh x$

$dx = du / \sinh x$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x} = \int \frac{1}{u} du = \ln u$

$= \ln \cosh x$

$If = e^{\int P dx} = e^{\ln \cosh x} \quad If = \cosh x$

$y \cdot If = \int Q \, If \, dx$

$y \cdot \cosh x = \int \sin 2x \, dx$

$y \cdot \cosh x = \frac{1}{2} \cdot -\cosh 2x + c$

$\cosh x \cdot y = \cosh 2x + c$

$y = \frac{\cosh 2x}{2} + c$

$\cosh x$

$y = \frac{\cosh 2x + 2c}{\cosh x}$

$\cosh 2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $dy/dx + 2y = e^{3x}$

$P = 2 \qquad \int P dx = 2x$

$Q = e^{3x}$

$If \; e^{\int P dx} = e^{2x}$

$y \cdot If = \int Q \cdot If \, dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} dx$

$y \cdot e^{2x} = \frac{1}{5} e^{5x} + C$

$y = \frac{\frac{1}{5} e^{5x} + c}{e^{3x}}$

c) $2 dy/dx = x^2 + 2x - 3$

$dy/dx = x + 2 - 3/x$

$\therefore \int dy/dx = \int x + 2 - 3/x \, dx$

$y = x^2/2 + 2x - 3\ln x + c$

d) $dy/dx + y/x = y^3$

$dy/dx \, y^{-3} + y^{-2}/x = 1 \quad -- (1)$

$z = y^{1-n} \qquad n = 3$

$z = y^{1-3} = z = y^{-2} \quad -- (2)$

$dz/dy = 2y^{-3} \, dy/dx \quad -- (3)$

Multiplying (eqn) (and $y^{(1-n)}$

$-2y^{-3} \frac{dy}{dx} - 2y^{-2}/x = -2$

and $dx/dy = -2y^{-3} \, dy/dx$

sub eqn (11) and (14) into (10)

$dx/dy - 2x/x = -2$

$\therefore P = -2/x \qquad Q = -2$

$\int P dx = -2\ln x$

$If = e^{-2\ln x} = x^{-2}$

$z \cdot If = \int Q \cdot If \cdot dx$

$z \cdot x^{-2} = \int -2x^{-2} dx$

$= -2x^{-1} + c$

$2x^{-1} = 2x^{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$y^2 = x(2+cx)$

$1/y^2 = x(2+cx)$

$y^2 = 1/x(2+cx)$

$y = \sqrt{1/x(2+cx)}$

e) $x^2 dy/dx = x^3 \sin 3x + 4$

$dy/dx = x \sin 3x + 4$

$\int dy/dx = \int x^1 \sin 3x + \int 4x^{-2}$

$= 1/3 \cos 3x + \int 1/3 \cos 3x + 4x^{-1}$

$= \frac{-x \cos 3x}{4x^{-1}/3} + \frac{\sin 3x}{9} -$

$y = \frac{\sin 3x}{4} - \frac{x \cos 3x}{3} - \frac{4}{5c}$

f). $(x^3 + xy^2)\, dy/dx = 2y^3$

$y = vx$

$dy/dx = v + x\, dv/dx$

$\frac{dy}{dx} = \frac{2y^3}{(x^3 + xy^2)}$

$= v + x\frac{dv}{dx} = \frac{2(vx)^3}{x^3 + x(vx)^2)}$

$v + x\frac{dv}{dx} = \frac{2(vx)^3}{(x^3 + x(vx)^2)}$

$v + x\frac{dv}{dx} = \frac{2v^3x^3}{x^3 + v^3x^3}$

$\therefore v + x\frac{dv}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$

$v + x\frac{dv}{dx} = \frac{2v^3}{1+v^2}$

$\frac{1+v^2}{v^3 - v}\, dv = \frac{1}{x}\, dx$

$\int \frac{1+v^2}{v^3 - v}\, dv = \int 1/x\, dx$

$= +\ln(v) + \ln(v-1) + \ln(v+1) \ln x + c$

$= \ln(y/x) + \ln(y/x - 1) + \ln(y/x + 1)$

$= \ln x + c$