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19/ENG05/067
Mechatronics
ENG 282

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a) $dy/dx = 2\sinh x - y\tanh x$

$dy/dx + y\tanh x = 2\sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P\,dx = \int \tanh x = \int \frac{\sinh x}{\cosh x}$

$\cosh x = u$

$\int \frac{\sinh x}{4}\,dx$

$y = \cosh x \quad dx = \frac{dy}{\sinh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2\int P\,dx = 2x$

$Q = e^{3x}$

$IF = e^{\int P\,dx} = e^{2x}$

$y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot e^{2x} = \int e^{3x} e^{2x}\,dx$

$y \cdot e^{2x} = \int e^{5x}\,dx$

$y \cdot e^{2x} = \frac{1}{5}e^{5x} + C$

$y = \frac{\frac{1}{5}e^{5x} + C}{e^{2x}}$

B) $\frac{dy}{dx} = \sinh x$

$\int \sinh x/4 \cdot dy/\sinh x$

$\int \frac{1}{4}du = \frac{1}{4} = \ln\cosh x$

$IF = e^{\int P\,dx} = e^{\ln\cosh x}$

$IF = \cosh x$

Then $y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x\,dx$

$2\sin x\cos x = \sin(2x)$

$2\sinh x\cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x\,dx$

$y \cdot \cosh = \cosh 2x + C$

$\cosh xy = \cosh 2x + C$

$y = \frac{\cosh 2x/2 + C}{\cosh x}$

$y = \frac{\cosh 2x + 2C}{\cosh x}$

Let $2C = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

c) $2\frac{dy}{dx} = x^{-\frac{1}{2}} + 2x - 5$

$\frac{dy}{dx} = x + 2 - \frac{5}{2}x$

$\int \frac{dy}{dx} = \int x + 2 \cdot \frac{3}{2}\,dx$

$y = \frac{x^3}{2} + 2x - 3\ln x + C$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$dy/dx \cdot y^3 + y^{-3}/x = 1$

$z = y^{1-n} \quad n = 3$

$z = y^{1-3}, z = y^{-2}$

$dy/dx = -2y^{-3}\,dy/dx$

Then multiply $-2y^{-3}\frac{dy}{dx} - \frac{2y^{-2}}{x} = -2$

and $dx/dy = -2y^{-3}$

$dx/dy - 2x/x = -2$

$P = -2/x, \quad Q = -2$

$\int P\,dx = -2\ln x$

$IF = e^{-2\ln x} = x^{-2}$

$2 \cdot IF = \int Q \cdot IF \, dx$

$2 \cdot x^{-2} = \int -2x^{-2} dx$

$= \frac{-2x^{-1}}{-1} + C$

$2x^{-2} = 2x^{-1} + C$

$2 = \frac{2x^{-1}}{x^{-2}} + \frac{C}{x^{-2}}$

$2 = 2x + Cx^2$

$2 = x(2 + Cx)$

$2 = y^{-2}$

$y^{-2} = x(2 + Cx)$

$y^{-2} = x(2 + Cx)$

$y^2 = \frac{1}{x}(2 + Cx)$

$y = \sqrt{\frac{1}{x}(2 + Cx)}$

e) $x^2 \, dy/dx = x^3 \sin 3x + 4$

$dy/dx = x \sin 3x + 4$

$\int dy/dx = \int x^4 \sin 3x + \int 4x^{-2}$

$= \frac{1}{3}\cos 3x + \int \frac{1}{3}\cos 3x + 4x^{-1}$

$= -x\cos 3x/3 + \sin 3x/9 - 4x$

$y = \sin 3x/4 - x\cos 3x/3 - 4/x$

F) $(x^3 + xy^2) \, dy/dx = 2y^3$

$y = dx$

$dy/dx = y + x \, dv/dx$

$v + x \frac{dv}{dx} = 2(vx)^3/x^3 + v^2x^3$

$v + x \, dv/dx = x^3(2v^3)/1 + v^2$

$P(\frac{dv}{dx} = 2v^3/1 + v^2$

$\frac{-2y^3 - v(1+v^2)}{1+v^2}$

$= \frac{2v^3 - v - v^3}{1+v^2}$

$x \, dv = v^3$

$\frac{1+v^2}{v^3 - v} \, dv + \frac{1}{x} dx$

$v(v-1)(v+1) = v^3 - v$

$\frac{1+v^2}{v^3-v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$1 + v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1), \; v = 1$

$1 + B = B(1)(2)$

$2 = B(2)$

$\therefore B = 1$

$v = -1$

$1 + (-1)^2 = ((-1)(-1-1)$

$\therefore C = 1$

$v = 0$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = A - 1$

$\therefore A = -1$

$\int \left[\frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right] dv =$

$\int \frac{-1}{v} dv - 1 \int \frac{1}{v-1} + dv + \int \frac{1}{v+1} dv =$

$\int \frac{1}{x} dv$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + C$

$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$

$\frac{v^2-1}{v} = Ax, \; y = vx \therefore v = \frac{y}{x}$

$$\frac{(y/x)^{2-1}}{(y/x)} = Ax$$

$$y^2/x^2 - 1 = Ax \cdot \frac{y}{x}$$

$$\frac{y^2}{x^2} - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$y^2 - x^2 = Ayx^2$$

$$y^2 = Ax^2y + x^2$$

$$y^2 = x^2(Ayx + 1)$$