$$\int \left(\frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right) du = \int \frac{1}{x} \cdot dx$$

$$-\ln v + \ln(v-1) + \ln(v+1) = \ln x$$

$$\ln (v-1)(v+1) - \ln v = \ln x$$

$$\frac{v^2 - 1}{v} = \ln x$$

$$y = vx$$

$$v = y/x$$

$$(y/x)^2 - 1 = Ax$$

$$y^2/x^2 - 1 = Ax - y/x$$

$$y^2/x^2 - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$y^2 - x^2 = Ay\,x^2$$

$$y^2 = Ax^2 y + x^2$$

$$y^2 = x^2(Ay + 1)$$

sub eqn (2) into (4)

$dz/dy - 2z/x = -2$

$\therefore P = -2/x \quad Q = -2$

$\int p \cdot dx = -2 \ln x$

$IF = e^{-2\ln x}$

$= x^{-2}$

$z \cdot IF = \int Q \cdot IF \cdot dx$

$z\, x^{-2} = \int -2x^{-2} \cdot dx$

$z x^{-2} = \frac{-2x^{-1}}{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$y^{-2} = x(2 + cx)$

$1/y^2 = x(2 + cx)$

$y^2 = 1/x(2 + cx)$

ⓑ $x^2 dy/dx = x^3 \sin 3x + 4$

$dy/dx = x \sin 3x + 4/x^2$

$\int dy/dx = \int x \sin 3x + \int 4x^{-2}$

$= -\frac{1}{3}x\cos 3x + \int \frac{1}{3}\cos 3x +$

$\frac{4x^{-1}}{-1}$

$= \frac{-x\cos 3x}{3} + \frac{\sin 3x}{4} - 4x^{-1}$

$y = \frac{\sin 3x}{4} - \frac{x\cos 3x}{3} - 4/x$

ⓒ $(x^3 + xy^2)\, dy/dx = 2y^3$

$y = vx$

$dy/dx = v + x\, dv/dx$

$v + x\, dv/dx = \frac{2(vx)^3}{x^3 + v^2x^3}$

$v + x\, dv/dx = \cancel{x^3}/\cancel{x^3}\left(\frac{2v^3}{(1+v^2)}\right)$

$= \frac{2v^3 - v(1+v^2)}{1+v^2}$

$= \frac{2v^3 - v - v^3}{1+v^2}$

$x \cdot dv = v^3$

$\frac{1+v^2}{v^3 - v} \cdot dv = 1/x \cdot dx$

$v(v-1)(v+1) = v^3 - v$

$\frac{1+v^2}{v^3 - v} = A/v + B/(v-1) + C/(v+1)$

$1 + v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$

$1 + 1^2 = B(1)(2)$

$2 = 2B$

$B = 1$

$v = -1$

$1 + (-1)^2 = C(-1)(-1-1)$

$2 = 2C$

$C = 1$

$v = 0$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = -1A$

$A = -1$

[8] EN...

$\frac{dy}{dx} = 2\sinh x - y\tanh x$

$\frac{dy}{dx} + y\tanh x = \sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P\,dx = \int \tanh x = \int \frac{\sinh x}{\cosh x}\,dx$

$\cosh x = u$

$\int \frac{\sinh x}{u}\,dx$

$u = \cosh x$

$du/dx = \sinh x$

$dx = du/\sinh x$

$\int \frac{\sinh x}{u} \cdot du/\sinh x$

$= \int 1/u \cdot du$

$= \ln u$

$= \ln \cosh x$

$If = e^{\int P\,dx}$

$= e^{\int \ln \cosh x}$

$= \cosh x$

$y \cdot If = \int Q \cdot If \cdot dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x\,dx$

$2\sinh x \cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x \cdot dx$

$y \cdot \cosh x = 1/2 \times 2\cosh 2x + c$

$y \cdot \cosh x \cancel{y} = \cosh 2x + c$

$y = \frac{\cosh 2x + c}{\cosh x}$

(b) $dy/dx + 2y = e^{3x}$

$P = 2$

$\int P\,dx = 2x$

$Q = e^{3x}$

$If = e^{\int P\,dx}$

$= e^{2x}$

$y \cdot If = \int Q \cdot If \cdot dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}\,dx$

$y \cdot e^{2x} = \int e^{5x}\,dx$

$y \cdot e^{2x} = 1/5\, e^{5x} + c$

$y = \frac{1/5\, e^{5x} + c}{e^{2x}}$

(c) $x\,dy/dx = x^2 + 2x - 3$

$dy/dx = x + 2 - 3/x$

$\int dy/dx = \int x + 2 - 3/x\,dx$

$y = x^2/2 + 2x - 3\ln x + c$

(d) $dy/dx + y/x = y^3$

$dy/dx\, y^{-3} + y^{-2}/x = 1$ ... (i)

$z = y^{1-n}$

$z = y^{1-3}$

$z = y^{-2}$ ... (ii)

$dz/dy = -2y^{-3}\,dy/dx$ ... (iii)

multiply equ by $(1-n)$

$-2y^{-3}\,dy/dx - 2y^{-3}/x = -2$

$dz/dy = -2y^{-3}\,dy/dx$

sub eqn (i) into ...

$dz/dy - 2z/x$ ...

$\therefore P = -2/x$

$\int P\,dx = -$...

$If = e^{-2\ln x}$

$= -2$

$z \cdot If = \int Q$...

$z\,x^{-2} = \int$...

$z\,x^{-2} = -$...

$z =$ ...

$z = 2x$...

$z = xc$...

$z = y^{-2}$

$y^{-2} =$ ...

$1/y^2$

y^2

(e) $x^2\,dy/(dx)$...

dy/dx...

$\int dy/dx$...

$= -x$...

$y = \frac{\sin...}{4}$...