NAME; ALE-ALABA OLUWASEUN OLUMIDE
MATRIC NO; 19/ENG06/064
DEPT; MECHANICAL

1) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

Solving using Integrating factor

$I\cdot F = e^{\int P dx}$

$I\cdot F = e^{\int \tanh x \, dx}$

$I\cdot F = e^{\int \frac{\sinh x}{\cosh x} dx} = e^{\ln(\cosh x)}$

$I\cdot F = \cosh x$

$y \cdot IF = \int Q \cdot IF$

$y \cdot \cosh x = \int 2\sinh x \cosh x$

Integrating R-H-S

$\int 2\sinh x \cosh x$

Let $u = \cosh x$

$\frac{du}{dx} = \sinh x \quad = \cancel{dx} = \frac{du}{\sinh x}$

$\int 2\cancel{\sinh x} \cdot u \cdot \frac{du}{\cancel{\sinh x}}$

$\int 2u \, du$

$= \frac{\cancel{2}u^2}{\cancel{2}}$

$= u^2 = \cosh^2 x$

Therefore for the full equation we have

$y \cdot \cosh x = \cosh^2 x + c$

$y = \frac{\cosh^2 x}{\cosh x} + \frac{c}{\cosh x}$

$y = \cosh x + c\cosh^{-1} x$

Scanned by CamScanner

Question 2

$$\frac{dy}{dx} + 2y = e^{3x}$$

Solving using Integrating factor

$$I.F = e^{\int P dx} = e^{\int 2 dx} = e^{2x}$$

$$y \cdot IF = \int Q \cdot IF$$

$$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}$$

$$y e^{2x} = \int e^{5x}$$

$$y e^{2x} = \frac{e^{5x}}{5} + c$$

$$y = \frac{e^{3x}}{5} + \frac{c}{e^{2x}}$$

Question 3

$$x\frac{dy}{dx} = x^2 + 2x - 3$$

Solving using direct integration

$$\frac{dy}{dx} = \frac{x^2}{x} + \frac{2x}{x} - \frac{3}{x}$$

$$\frac{dy}{dx} = x + 2 - \frac{3}{x}$$

$$\int dy = \int \left(x + 2 - \frac{3}{x}\right) dx$$

$$y = \int \left(x + 2 - \frac{3}{x}\right) dx$$

$$y = \frac{x^2}{2} + 2x - 3 \ln x + C$$

Scanned by CamScanner

Question 4

$$\frac{dy}{dx} + \frac{y}{x} = y^3$$

Solving using Bernoulli

$$Z = y^{1-n} \quad Z = y^{1-3} = y^{-2}$$

$$\frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx} = 1-3y^{-3}\frac{dy}{dx} = -2y^{-3}\frac{dy}{dx}$$

from Main equation

$$\frac{dy}{dx} + \frac{y}{x} = y^3$$

Dividing through by y^3

$$y^{-3}\frac{dy}{dx} + \frac{1}{xy^2} = 1$$

Multiplying through by -2

$$-2y^{-3}\frac{dy}{dx} - \frac{2}{xy^2} = -2$$

$$-2y^{-3}\frac{dy}{dx} - \frac{2}{x}y^{-2} = -2$$

$$\frac{dz}{dx} - \frac{2}{x}z = -2$$ → Solving using Integrating factor

$$I.F = e^{\int \frac{-2}{x} dx} = e^{-2\ln x} = x^{-2}$$

$$x^{-2} \cdot Z = \int -2 \cdot x^{-2}$$

$$Zx^{-2} = \frac{-2x^{-2+1}}{-2+1}$$

$$Zx^{-2} = 2x^{-1} + C$$

Recall that $Z = y^{-2}$

$$y^{-2}x^{-2} = 2x^{-1} + C$$

$$y^2 = \frac{x^{-2}}{2x^{-1} + C}$$

$$y^2[2x^{-1} + C] = x^{-2}$$

Dividing through by x^{-2} we have

$$y^2x^2[2x^{-1} + C] = 1$$

Scanned by CamScanner

$$y^2x^2 2x^{-1} + y^2x^2C = 1$$
$$y^2x\,2 + y^2x^2C = 1$$
$$xy^2[cx + 2] = 1$$

Question (5)

$$x^2\frac{dy}{dx} = x^3\sin 3x + 4$$

Solving by direct Integration

$$\frac{dy}{dx} = x\sin 3x + \frac{4}{x^2}$$

$$\int dy = \int\left(x\sin 3x + \frac{4}{x^2}\right)dx$$

$$y = \int x\sin 3x + \int\frac{4}{x^2}$$

Integrating $x\sin 3x$ using Integration by Parts

$$\int \overset{u}{x}\,\overset{dv}{\sin 3x} \quad \text{and} \quad \int u\,dv = uv - \int v\,du$$

$$u = x \qquad du = 1$$

$$dv = \sin 3x \qquad v = \frac{-\cos 3x}{3}$$

$$\frac{-x\cos 3x}{3} - \int\frac{-\cos 3x}{3}\cdot 1$$

$$= \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9}$$

Placing everything together

$$y = \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} + \frac{4x^{-2+1}}{-2+1}$$

$$y = \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} - \frac{4}{x} + C$$

Question (6)

$$\left(x^3 + xy^2\right)\frac{dy}{dx} = 2y^3$$

Solving using homogeneous Method

$y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{2y^3}{x^3 + xy^2}$$

$$\frac{dy}{dx} = \frac{2(vx)^3}{x^3 + x(vx)^2}$$

$$\frac{dy}{dx} = \frac{x^3(2v^3)}{x^3(1 + v^2)}$$

$$\frac{dy}{dx} = \frac{2v^3}{1+v^2}$$

$$v + x\frac{dv}{dx} = \frac{2v^3}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{2v^3}{1+v^2} - v$$

$$x\frac{dv}{dx} = \frac{2v^3 - v(1+v^2)}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{2v^3 - v - v^3}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$$

Separating variables we have

$$\int \frac{1+v^2}{v^3 - v}\,dv = \int \frac{1}{x}\,dx$$

Integrating L.H.S using partial fractions

$$\frac{1+v^2}{v^3 - v} = \frac{1+v^2}{v(v^2-1)} = \frac{1+v^2}{(v)(v+1)(v-1)}$$

$$\frac{1+v^2}{(v)(v+1)(v-1)} = \frac{A}{v} + \frac{B}{v+1} + \frac{C}{v-1}$$

$$1+v^2 = A(v+1)(v-1) + B(v)(v-1) + C(v)(v+1)$$

When $V = 1$

$2 = A(0) + B(0) + C(1)(2)$

$2 = 2C$

$C = 1$

When $V = -1$

$2 = A(0) + B(-1)(-2) + C(0)$

$2 = 2B$

$B = 1$

When $V = 0$

$1 = A(1)(-1) + B(0) + C(0)$

$A = -1$

therefore we have

$$\int \frac{-1}{v} + \int \frac{1}{v+1} + \int \frac{1}{v-1}$$

$$-\ln(v) + \ln(v+1) + \ln(v-1)$$

Joining everything together we have

$-\ln v + \ln(v+1) + \ln(v-1) = \ln x + c$

Let $c = \ln A$

$$\ln\left(\frac{(v+1)(v-1)}{v}\right) = \ln x + \ln A$$

$$\ln\left(\frac{(v+1)(v-1)}{v}\right) = \ln(Ax)$$

~~$(v+1)$~~ Taking exponents of both sides

$$\frac{(v+1)(v-1)}{v} = Ax$$

Recall $v = \frac{y}{x}$

$$\frac{v^2-1}{v} = Ax$$

$$\frac{\frac{y^2}{x^2} - 1}{y/x} = Ax$$

$$\frac{\left(\frac{y^2}{x^2} - 1\right)}{\frac{y}{x}} = Ax$$

$$\frac{x\left(\frac{y^2}{x^2} - 1\right)}{y} = Ax$$

$$\frac{\left(\frac{y^2}{x} - x\right)}{y} = Ax$$

$$\frac{y^2}{x} - x = Axy$$

$$y^2 - x^2 = Ax^2 y$$

$$\therefore \quad y^2 - x^2 = Ax^2 y$$