

$$\frac{dz}{dy} - \frac{dz}{dx} = -2$$

$$\therefore P = -\frac{1}{x}, Q = -2$$

$$2 \cdot H = \int Q \cdot H dx$$

$$2 \cdot x^{-2} = \int -2x \cdot 2 dx$$

$$2 = \frac{-2x^{-1}}{-1x^{-2}} + C$$

$$2x^{-2} = 2x^{-1} + C$$

$$2 = \frac{2x^{-1}}{x^{-2}} + \frac{C}{x^{-2}}$$

$$2 = 2x + Cx^2$$

$$2 = x(2 + Cx)$$

$$2 = y^{-2}$$

$$y^{-2} = x(2 + Cx)$$

$$\frac{1}{y^2} = x(2 + Cx)$$

$$y^2 = \frac{1}{x(2 + Cx)}$$

$$\therefore y = \frac{1}{\sqrt{x(2 + Cx)}}$$

$$y = \frac{1}{\sqrt{x(2 + Cx)}}$$

$$e) x^2 \frac{dy}{dx} = x^2 \sin 3x + 4$$

$$\frac{dy}{dx} = x \sin 3x + 4$$

$$\int \frac{dy}{dx} = \int x^4 \sin 3x + \int 4x^{-2}$$

$$= \frac{1}{3} \cos 3x + \int \frac{1}{3} \cos 3x + \frac{4x^{-1}}{-1}$$

$$= -\frac{x}{3} (\cos 3x + \frac{\sin 3x}{3} - \frac{4x^{-1}}{-1})$$

$$= -\frac{x}{3} (\cos 3x + \frac{\sin 3x}{3} - \frac{4x^{-1}}{-1})$$

$$y = \frac{\sin 3x}{3} - \frac{x \cos 3x}{3} - \frac{4}{x}$$

$$3) (x^3 + xy^2) \frac{dy}{dx} = 2y^3$$

$$y = \sqrt{x}$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = 2(v^2)^3$$

$$v + x \frac{dv}{dx} = \frac{2x^3 (9x^3)}{x^3 (1+v^2)}$$

$$= \frac{2v^3 - v(1-v^2)}{1+v^2}$$

$$= \frac{2v^3 - v - v^3}{1+v^2}$$

$$x \frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$$

$$\frac{1+v^2}{v^3 - v} dv = \frac{dx}{x}$$

$$v(v-1)(v+1) = v^3 - v$$

$$\frac{1+v^2}{v^3 - v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$$

$$1+v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$$

$$v=1$$

$$1+1^2 = B(1)(2)$$

$$2 = B(2)$$

$$\therefore B = 1$$

$$v = -1$$

$$1 + (-1)^2 = C(-1)(-1-1)$$

$$2 = C(-2)$$

$$\therefore C = -1$$

$$v = 0$$

$$1 + (0)^2 = A(0-1)(0+1)$$

$$1 = A(-1)(1)$$

$$2 = A-1$$

$$\therefore A = -1$$

$$\int \left[-\frac{1}{v} + \frac{1}{v+1} + \frac{1}{v-1} \right] dv = \int \frac{dx}{x}$$

$$\int \frac{dx}{x}$$

$$\int \frac{1}{v} dv + \int \frac{1}{v-1} dv + \int \frac{1}{v+1} dv = \int \frac{1}{x} dx$$

$$= \int \frac{1}{x} dx$$

$$\begin{aligned} & \textcircled{1} -\ln v + \ln(v-1) + \ln(v+1) + C \\ & = \ln(v-1)(v+1) - \ln v = \ln x \\ & = \frac{y^2-1}{v} = Ax \end{aligned}$$

$$v = \frac{y}{x} \therefore x = \frac{y}{v}$$

$$\left(\frac{y}{x}\right)^2 - 1 = Ax$$

$$\frac{y^2}{x^2} - 1 = Ax - \frac{y}{x}$$

$$\frac{y^2}{x^2} + 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$\frac{y^2 - x^2}{x^2} \cdot y^2 - x^2 = Ayx^2$$

$$y^2 = Ax^2y + x^2$$

$$y^2 = x^2(Ay + 1) \parallel$$

a) $\frac{dy}{dx} = 2 \sinh x - y \tanh x$

$\frac{dy}{dx} + y \tanh x = 2 \sinh x$

$P = \tanh x$

$Q = 2 \sinh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$
 $\cosh x = u$

$\int \frac{\sinh x}{u} dx$

$u = \cosh x \quad dx = \frac{du}{\sinh x}$

$\frac{du}{dx} = \sinh x$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x}$

$\int \frac{1}{u} du = \ln u = \ln \cosh x$

$I.F = e^{\int P dx} = e^{\ln \cosh x}$

$I.F = \cosh x$

$I.F = \cosh x$

then $y \cdot I.F = \int Q \cdot I.F dx$

$y \cdot \cosh x = \int 2 \sinh x \cdot \cosh x dx$

$2 \sinh x \cosh x = \sinh(2x)$

$2 \sinh x \cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x dx$

$\cosh x y = \cosh 2x + c$

$y = \frac{\cosh 2x + c}{\cosh x}$

$\cosh x$

$y = \frac{\cosh 2x + 2c}{\cosh x}$

$\cosh x$

let $2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2 \quad \int P dx = 2x$

$Q = e^{3x}$

$I.F = e^{\int P dx} = e^{2x}$

$\therefore y \cdot I.F = \int Q \cdot I.F dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} dx$

$y \cdot e^{2x} = \int e^{5x} dx$

$y \cdot e^{2x} = \frac{1}{5} e^{5x} + c$

$y = \frac{\frac{1}{5} e^{5x} + c}{e^{2x}}$

c) $2 \frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\therefore \int \frac{dy}{dx} = \int x + 2 - \frac{3}{x} dx$

$y = \frac{x^2}{2} + 2x - 3 \ln x + c$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$\frac{dy}{dx} y^{-3} + y^{-4} = 1 \dots (1)$

$z = y^{1-n} \quad n=3$

$z = y^{1-3}, \quad z = y^{-2} \dots (2)$

$\therefore \frac{dz}{dy} = -2y^{-3} \frac{dy}{dx} \dots (3)$

then multiply equ (1) by $1-n$

$-2y^{-3} \frac{dy}{dx} - \frac{2y^{-3}}{x} = -2$

Subs and $\frac{dz}{dy} = \frac{-2y^{-3} dy}{dx}$

Sub eq 2 into 4