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Chemical Engineering

1. $dy/dx + y \tanh x = 2 \sinh x$

$P = \tanh x$

$Q = 2 \sinh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} \cdot dx$

$= \ln \cosh x$

$IF = e^{\int P dx} = e^{\ln \cosh x}$

$y \cdot IF = \int Q \cdot IF \, dx$

$y \cdot e^{\cancel{\ln \cosh x}} = \int 2 \sinh x \cdot e^{\cancel{\ln \cosh x}} \cdot dx$

$y = \int 2 \sinh x \, dx = y = 2 \cosh x + c$

2. $dy/dx + 2y = e^{3x}$

$P = 2$

$Q = e^{3x}$

$\int P dx = 2x$

$IF = e^{\int P dx} = e^{2x}$

$y \cdot IF = \int IF \cdot Q dx$

$y \cdot e^{2x} = \int e^{2x} \cdot e^{3x} dx$

$ye^{2x} = \int e^{5x} dx$

$ye^{2x} = \frac{1}{5} e^{5x} + c$

$y = \frac{1}{5} e^{3x} + ce^{-2x}$

3. $x \, dy/dx = x^2 + 2x - 3$

$dy/dx = x + 2 - 1/x \cdot 3$

$dy = (\cancel{x} + 2 - 3/x) dx$

$\int dy = \int (x + 2 - 3 1/x) dx$

$y = \frac{1}{2} x^2 + 2x - 3 \ln x + c$

4. $dy/dx + y/x = y^3$

$y^{-3} dy/dx + y^{-2} x^{-1} = 1$

$z = y^{1-3} = y^{-2} = y^{-2}$

$dz/dx = -2y^{-3} dx$

$-2y^{-3} dy/dx + (-2y^{-2} x^{-1}) = -2$

$dz/dx - 2x^{-1} z = -2$

$P = -2x^{-1}$

$Q = -2$

$\int P dx = -2 \ln x$

$IF = e^{\int P dx} = 1/x^2 \rightarrow z \cdot 1/x^2 = \int 1/x^2 \cdot -2 dx$

$y = (2x + cx^2)^{-1/2} = y$

5. $x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$

$\frac{dy}{dx} = x \sin 3x + 4x^{-2}$ ~~dx~~

$\Rightarrow dy = x \sin 3x + 4x^{-2} dx$

$y = \int x \sin 3x + 4x^{-2} dx$

Integrate by parts for $\int x \sin 3x$

$\int u dv = uv - \int v du$ $\quad u = x \quad dv = \sin 3x \quad du = 1 \quad v = -\frac{1}{3}\cos 3x$

$= -\frac{1}{3} x \cos 3x - \int -\frac{1}{3} \cos 3x$

$= -\frac{1}{3} x \cos 3x + \frac{1}{3}\left(\frac{1}{3} \sin 3x\right)$

$\int 4x^{-2} dx = \frac{4x^{-3}}{-3} = -\frac{4}{3} x^{-3}$

$y = -\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x - \frac{4}{3x^3}$

6. $(x^3 + xy^2) \frac{dy}{dx} = 2y^3$

$\frac{dy}{dx} = \frac{2y^3}{(x^3 + xy^2)}$

$\cancel{x} + x\frac{dv}{dx} = \frac{2v^3x^3}{(x^3 + v^2x^3)}$

$= \frac{\cancel{x^3}\, 2v^3}{\cancel{x^3}(1 + v^2)} = \frac{2v^3}{(1+v^2)}$

$x \frac{dv}{dx} = \frac{2v^3}{(1+v^2)} - \frac{v}{1}$

$x \frac{dv}{dx} = \frac{2v^3 - v - v^3}{(1+v^2)(1)} = \frac{v^3 - v}{1+v^2} = \frac{v(v^2-1)}{1+v^2}$

$\frac{1+v^2}{v(v+1)(v-1)} dv = \frac{1}{x} dx$

Integrating by partial fraction

$\frac{1+v^2}{v(v+1)(v-1)} = \frac{A}{v} + \frac{B}{v+1} + \frac{C}{v-1}$

$A = -1 \quad B = 1 \quad C = 1$

$\int -\frac{1}{v} + \int \frac{1}{v+1} + \int \frac{1}{v-1} = \int \frac{1}{x} dx$

$-\ln v + \ln(v+1) + \ln(v-1) = \ln x + C$ $\qquad$ let $C = \ln A$

$-\ln v + \ln(v+1) + \ln(v-1) = \ln x + \ln A$

$\ln\left(\frac{(v+1)(v-1)}{v}\right) = \ln(Ax)$

$\frac{v^2 - 1}{v} = Ax$ $\qquad$ since $v = y/x$

$v - \frac{1}{v} = Ax \quad = \quad \frac{y}{x} - \frac{x}{y} = Ax$

$\frac{y}{x} - \frac{x}{y} = Ax$