CHIMA WISDOM ENYICHE
18/ENG04/025
Electrical /Electronics
ENG 282: ENGineering mathematics II

Solve the following

1) $\frac{dy}{dx} + y \tanh x = 2\sinh x$

$y \cdot IF = \int Q ~~IF~~ \, dx$

$I.F = e^{\int P dx}$

$P = \tanh x$

$\int P dx = \ln(\cosh x)$

$I.F = e^{\ln \cosh x} = \cosh x$

$\therefore \; y \cdot \cosh x = \int 2\sinh x \cdot \cosh x \, dx$

$y \cdot \cosh x = 2\int \sinh x \cdot \cosh x$

$y \cdot \cosh x = 2\left[\frac{\sinh^2 x}{2}\right] + c$

$y = \frac{2\sinh^2 x}{2\cosh x} + \frac{c}{\cosh x}$

$y = (\tanh x)\sinh x + \frac{c}{\cosh x}$

2) $\frac{dy}{dx} + 2y = e^{3x}$

$y \cdot IF = \int Q \cdot IF \, dx$

$P = 2$

$\int P \, dx = 2x$

$I.F = e^{\int P dx} = e^{2x}$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} \, dx$

$y \cdot e^{2x} = \int e^{5x} \, dx$

$y \cdot e^{2x} = \frac{e^{5x}}{5} + c$

$y = \frac{e^{5x}}{5e^{2x}} + ce^{-2x}$

$$y = \frac{e^{3x}}{5} + ce^{-2x}$$

(3)

$$x\frac{dy}{dx} = x^2 + 2x - 3$$

$$\frac{dy}{dx} = \frac{x^2}{x} + \frac{2x}{x} - \frac{3}{x}$$

$$\frac{dy}{dx} = x + 2 - \frac{3}{x}$$

$$\int dy/dx = \int (x + 2 - 3/x)$$

$$y = \frac{x^2}{2} + 2x - 3\ln x + c$$

4)

$$\frac{dy}{dx} + \frac{y}{x} = y^3$$

$$y^{-3}\frac{dy}{dx} + \frac{y}{y^3 x} = 1$$

$$y^{-3}\frac{dy}{dx} + \frac{1}{y^2 x} = 1$$

$$z = y^{1-n}$$

$$z = y^{1-3}$$

$$z = y^{-2}$$

$$\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx} \quad \cdots (1)$$

$$-2y^{-3}\frac{dy}{dx} - 2\frac{y^{-2}}{x} = -2$$

$$\frac{dz}{dx} - \frac{2}{x}z = -2$$

$$z \cdot IF = \int Q \cdot IF\, dx$$

$$P = 2/x \quad \int P dx = 2\ln x$$

$$e^{2\ln x} = x^2$$

$$z \cdot x^2 = \int -2 \cdot x^2\, dx$$

$$z \cdot x^2 = \frac{-2x^3}{3} + c$$

$$z = \frac{-2x}{3} + \frac{c}{x^2}$$

$$\frac{1}{y^2} = \frac{-2x}{3} + \frac{c}{x^2}$$

e) $x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$

$$\frac{dy}{dx} = x \sin 3x + \frac{4}{x^2}$$

$$\int \frac{dy}{dx} = \int \left(\overset{u}{x} \overset{dv}{\sin 3x} + \frac{4}{x^2} \right)$$

$$y = ~~uv - \int u~~ \quad uv - \int v\,du$$

$+(\frac{1}{x^2}$

$+(x^{-2})$

$$\frac{-x \cos 3x}{3} - \int \frac{\cos 3x}{3}$$

$\frac{x^{-1}}{-1} \quad -x^{-1}$

$$\frac{-x \cos 3x}{3} - \frac{1}{3} \int \cos 3x$$

$$\frac{-x \cos 3x}{3} - \frac{1}{3} \frac{\sin 3x}{3}$$

$$\frac{-x \cos 3x}{3} - \frac{\sin 3x}{9} + (-4x^{-1}) + c$$

$$y = \frac{-x \cos 3x}{3} - \frac{\sin 3x}{9} - \frac{4}{x} + c$$

4) $(x^3 + xy^2) \frac{dy}{dx} = 2y^3$

$$\frac{dy}{dx} = \frac{2y^3}{(x^3 + xy^2)}$$

$y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{2y^3}{(x^3 + xy^2)}$$

$$v + x\,dv/dx = \frac{2(vx)^3}{x^3 + x(vx)^2}$$

$$v + x\frac{dv}{dx} = \frac{2v^3x^3}{x^3 + v^2x^3}$$

$$v + x\frac{dv}{dx} = \frac{2v^3x^3}{x^3(1+v^2)}$$

$$v + x\frac{dv}{dx} = \frac{2v^3}{(1+v^2)}$$

$$x\frac{dv}{dx} = \frac{2v^3}{(1+v^2)} - v$$

$$x\frac{dv}{dx} = \frac{2v^3 - v - v^3}{(1+v^2)}$$

$$x\frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$$

$$\frac{dv}{dx} = \frac{(v^3 - v)}{(1+v^2)x}$$

$$(v^3 - v)dx = (1+v^2)(x)\,dv$$

$$\frac{dx}{x} = \frac{(1+v^2)}{(v^3-v)}dv$$

$$\frac{dx}{x} = \frac{(1+v^2)}{v(v^2-1)}dv$$

using partial fractions

$$\frac{1+v^2}{v^3-v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$$

$$1+v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$$

$$1+1^2 = B(1)(2)$$

$$2 = 2B$$

$$B = 1$$

$$v = -1$$

$$1+(-1)^2 = C(-1)(-1-1)$$

$$2 = 2C$$

$$C = 1$$

$$v = 0$$

$$1+(0)^2 = A(0-1)(0+1)$$

$$1 = A(-1)(1)$$

$$1 = (-1)A \quad A = -1$$

$$\frac{dx}{x} = \frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1}\,dv$$

$$\ln x = -\ln v + \ln(v-1) + \ln(v+1) + c$$

let $c = \ln A$

$$\ln x = \ln(v-1) + \ln(v+1) - \ln v + \ln A$$

$$x = \frac{(v-1)(v+1)(A)}{v}$$

$$y = vx$$

$$v = \frac{y}{x}$$

$$\therefore \quad x = \frac{(y/x - 1)(y/x + 1)(A)(x)}{y}$$

$$(y/x - 1)(y/x + 1)(A)(x) = yx$$

$$(y/x - 1)(y/x + 1)(A) = y$$

$$\left(\frac{y^2}{x^2} - 1\right)(A) = y$$

$$Ay^2/x^2 - A = y$$

$$\frac{Ay^2 - Ax^2}{x^2} = y$$

$$Ay^2 - Ax^2 = yx^2$$

$$Ay^2 = yx^2 + Ax^2$$

$$Ay^2 = x^2(y + A)$$