BIOMEDICAL ENGINEERING

(a) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

$\frac{dy}{dx}$ Integ

Let $p = \tanh x$

$Q = 2\sinh x$

$\int p \cdot dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x$

$\frac{dy}{dx} = \sinh x$

$dx = \frac{du}{\sinh x}$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x}$

$\int \frac{1}{u} \cdot du$

$= \ln u$

$= \ln \cosh x$

$If = e^{\int p \cdot dx}$

$e^{\int \ln\cosh x}$

$= \cosh x$

$y \cdot If = \int Q \cdot If\, dx$

$y \cdot \cosh x = \int 2\sinh x - \cosh x\, dx$

$2\sinh x \cdot \cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x \cdot dx$

$y = \frac{\cosh 2x + c}{\cosh x}$

(B.) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2$

$\int p \cdot dx = 2x$

$Q = e^{3x}$

$If\ e^{\int p \cdot dx}$

e^{2x}

$y \cdot If = \int Q \cdot If \cdot dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} \cdot dx$

$y e^{2x} = \int e^{5x}\, dx$

$y = e^{2x} = 1/5\, e^{5x} + c$

$y = \frac{1/5 \cdot e^{5x} + c}{e^{2x}}$

(c). $x\frac{dy}{dx} = x^2 + 2x -$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\int \frac{dy}{dx} = \int x + 2 - \frac{3}{x}$

$y = \frac{x^2}{2} + 2x - 3\ln x +$

$\frac{dy}{dx} \quad x$

$\frac{dy}{dx}y^{-3} + \frac{y^{-2}}{x} = 1$ — ①

$z = y^{1-n}$

$z = y^{1-3}$

$z = y^{-2}$ — ②

$\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$ — ③

multiply equation by $(1-n)$

$-2y^{-3}\frac{dy}{dx} - \frac{2y^{-3}}{x} = -2$

$\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$ — ④

Substitute equation ② into ④

$\frac{dz}{dy} - \frac{2}{x} = -2$

$\therefore P = -2,\ Q = -2$

$\int p \cdot dx = -2\ln x$

$If = e^{-2 \ln x}$

$= -2$

$z \cdot If = \int Q \cdot If \cdot dx$

$2x^{-2} = \int 2x^{-2}\,dx$

$2x^{-2} = \frac{-2x^{-1}}{-1} + C$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{C}{x^{-2}}$

$z = 2x + Cx^2$

$z = x(2 + Cx)$

$y^{-2} = x(x + Cx)$

$\frac{1}{y^2} = x(2 + Cx)$

$y^2 = \frac{1}{x(2 + Cx)}$

ⓔ

$x^2\frac{dy}{dx} = x^2 \sin 3x + 4$

$\frac{dy}{dx} = x \sin x + \frac{4}{x^2}$

$\int \frac{dy}{dx} = \int x \sin x + \int 4x^{-2}$

$= \frac{-1}{3} \cdot \cos 3x + \int \frac{1}{3} \cos 3x$

$+ \frac{4x^{-1}}{-1}$

$= \frac{x \cos 3x}{3} + \frac{\sin 3x}{4} - 4x^{-1}$

$y = \frac{\sin 3x}{4} \cdot \frac{x \cos 3x}{3}$

$- \frac{4}{x}$

ⓕ $(x^3 + xy^2)\,dy/dx = 2y^3$

$y = Vx$

$\frac{dy}{dx} = V + x\frac{dy}{dx}$

$V + x\frac{dy}{dx} = \frac{x^3}{x^3}\left(\frac{2V^3}{1+V^2}\right)$

$= \frac{2V^3 - V(1 + V^2)}{1 + V^2}$

$\frac{...}{V^3-u} = \frac{1}{x}\cdot dx$

$V(V-1)(V+1) = V^3 - 4$

$\frac{1+V^2}{V^3-V} = \frac{A}{V} + \frac{B}{V-1} + \frac{C}{V+1}$

$1+V^2 = A(V-1)(V+1)$
$\quad + B(V)(V+1)$
$\quad + C(V)(V-1)$

$1+1^2 = B(1)(2)$
$2 = 2B$
$B = 1$
$V = -1$

$1 + (-1)^2 = C(-1)(-1-1)$
$2 = 2C$
$C = 1$
$V = 0$

$1 + (0)^2 = A(0-1)(0+1)$
$1 = A(-1)(1)$
$1 = -1A$
$A = -1$

$\int\left(\frac{-1}{V} + \frac{1}{V-1} + \frac{1}{V+1}\right)dv$

$= \int \frac{1}{4}\cdot dx$

$-\ln V + \ln(V-1) + \ln(V+1)$
$= \ln x \ln(V-1)(V+1)$
$-\ln V = \ln x$
$\frac{V^2-1}{V} = \ln x$

$y = Vx$
$V = \frac{y}{x}$

$\frac{y^2}{x^2} - 1 = Ax - \frac{y}{x}$

$\frac{y^2}{x^2} - 1 = Ay$

$\frac{y^2-x^2}{x^2} = Ay$

$\frac{y^2-x^2}{y^2} = Ay$

$y^2 = Ax^2y + x^2$

$y = x^2(Ay+1)$