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a) $dy/dx = 2\sinh x - y\tanh x$

$dy/dx + y\tanh x = \sin x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x = u$

$\int \frac{\sinh x}{u} dx$

$u = \cosh x \qquad dx = \frac{dy}{\sinh x}$

$dy/dx = \sinh x$

$\int 1/u \, du = \ln u = \ln \cosh x$

if $e^{\int P dx} = e^{\ln \cosh x}$

$I.F = \cosh x$

Then $y \, I.F = \int Q . I.F dx$

$y . \cosh x = \int 2\sinh x . \cosh x \, dx$

$2\sin x \cos x = \sin(2x)$

$2\sinh x \cosh x = \sinh(2x)$

$y . \cosh x = \frac{1}{2} . 2\cosh x + c$

$\cosh xy = \cosh 2x + C$

$y = \frac{\frac{\cosh 2x}{2} + c}{\cosh x}$

$y = \frac{\cosh 2x + 2c}{\cosh x}$

Let $2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $dy/dx + 2y = e^{3x}$

$P = 2 \quad \int P dx = 2x$

$Q = e^{3x}$

$If = e^{\int P dx} = e^{2x}$

$y . I.F = \int Q . I.F dx$

$y . e^{2x} = \int e^{3x} . e^{2x} dx$

$y . e^{2x} = \int e^{5x} dx$

$y . e^{2x} = 1/5 e^{5x} + c$

$y = \frac{1/5 e^{5x} + c}{e^{2x}}$

c) $2 \, dy/dx = x^2 + 2x - 5$

$dy/dx = x + 2 - 3/x$

$\therefore \int dy/dx = \int x + 2 - 3/x \, dx$

$y = x^2/2 + 2x - 3\ln x + c$

d) $dy/dx + y/x = y^3$

$dy/dx \, y^{-3} + y^{-3}/x = 1$ --- (i)

$z = y^{1-n}, n = 3$

$z = y^{1-3}, z = y^{-2}$ --- (ii)

$dz/dy = 2y^{-3} \, dy/dx$ --- (iii)

Thus multiplying eq(i) by $1-n$

$-2y^{-3} \, dy/dx - 2y^{-2}/x = -2$

and $dz/dy = -2y^{-3} dy/dx$

sub eqn (ii) & (iii) into (iv)

$dz/dy - 2x/x = -2$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + c$

$\ln(v-1)(v+1) - \ln v = \ln x + \ln A$

$\frac{v^2-1}{v} = Ax$

$y = vx \quad v = y/x$

$\frac{(y/x)^2 - 1}{y/x} = Ax$

$\frac{y^2}{x^2} - 1 = Ax \cdot y/x$

$y^2/x^2 - 1 = Ay$

$\frac{y^2 - x^2}{x^2} = Ay$

$y^2 - x^2 = Ayx^2$

$y^2 = Ax^2y + x^2$

$y^2 = x^2(Ay + 1)$

$\therefore P = -2/x, \quad Q = -2$

$\int P\,dx = -2\ln x$

$I.F = e^{-2\ln x} = x^{-2}$

$z \cdot I.F = \int Q.I.F.\,dx$

$z.x^{-2} = \int -2x^{-2}\,dx$

$= \frac{-2x^{-1}}{-1} + C$

$2x^{-1} = 2x^{-1} + C$

$z = \frac{2x^{-1}}{x^{-3}} + \frac{C}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2+x)$

$z = y^{-2}$

$y^{-2} = x(2+cx)$

$1/y^2 = x(2+cx)$

$y^2 = 1/x(2+cx)$

$\therefore y = \sqrt{1/x\, 2+cx}$

$y = 1/\sqrt{x(2+cx}$

e) $x^2\, dy/dx = x^3 \sin 3x + 4$

$dy/dx = x \sin 3x + 4/x^2$

$\int dy/dx = \int x^4 \sin 3x + \int 4x^{-2}$

$-\frac{1}{3}\cos 3x + \int \frac{1}{3}\cos 3x + 4x^{-1}$

$= -\frac{x\cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1}$

$y = \frac{\sin 3x}{4} - \frac{x\cos 3x}{3} - \frac{4}{x}$

f) $(x^3 + xy^3)\, dy/dx = 2y^3$

$y = vx$

$dy/dx = v + x\, dv/dx$

$v + x\, dv/dx = \frac{1(vx)^3}{x^3 + v^2x^3}$

$v + x\, dv/dx = \frac{x^3(2v)}{x^3(1+v^2)}$

$x\, dv/dx = \frac{2v^3}{1+v^2}$

$= \frac{2v^3 - v(1+v^2)}{1+v^2}$

$= \frac{2v^3 - v - v^3}{1+v^2}$

$x\, dv = v^3$

$\frac{1+v^2}{v^3 - v}\, dv = 1/x\, dx$

$v(v-1)(v+1) = v^3 - v$

$\frac{1+v^2}{v^3-v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$1+v^2 = A(v-1)(v+1) + B(v)(v+1)$

$+ C(v)(v-1), \; v-1$

$1+1^2 = B(1)(2)$

$2 = B2$

$\therefore B = 1$

$v = -1$

$1 + (-1)^2 = C(-1)(-1-1)$

$2 = 2c$

$\therefore C = 1$

$v = 0$

$1 + 0^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$\therefore A = -1$

$\int \left[\frac{1}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right] dv$

$= \int dx\, 1/2 \int -1/v\, dv +$

$\int 1/v-1 + dv + \int 1/v+1\, dv$

$= \int 1/x\, dy$

$-\ln v$

$\ln ($

$v^2 -$

v

y