TAIWO ADIBOLA EMMANUEL
19/ENG01/024
CHEMICAL ENGINEERING
ENG 282

Question 1

$\frac{\partial y}{\partial x} + y \tanh x = 2\sinh x$

Solving using Integrating factor

$I.F = e^{\int P \partial x}$

$I.F = e^{\int \tanh x \, \partial x}$

$I.F = e^{\int \frac{\sinh x}{\cosh x} \partial x} = e^{\ln(\cosh x)}$

$I.F = \cosh x$

$y \cdot I.F = \int Q \cdot IF \, \partial x$

$y \cdot \cosh x = \int 2\sinh x \times \cosh x$

Integrating R.H.S

$\int 2\sinh x \cosh x$

let $u = \cosh x$

$\frac{\partial u}{\partial x} = \sinh x \quad \partial x = \frac{\partial u}{\sinh x}$

$\int 2\sinh x \cdot u \cdot \frac{\partial u}{\sinh x}$

$\int 2u \, \partial u$

$= \frac{2u^2}{\cancel{2}}$

$u^2 = \cosh^2 x$

$y \cdot \cosh x = \cosh^2 x + c$

$y = \frac{\cosh^2 x}{\cosh x} + \frac{c}{\cosh x}$

$y = \cosh x + c \cosh^{-1} x$

Question 2

$$\frac{dy}{dx} + 2y = e^{3x}$$

Solving using Integrating factor

$$I.F = e^{\int P\partial x} = e^{\int 2du} = e^{2x}$$

$$y \cdot IF = \int Q \cdot IF$$

$$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}$$

$$ye^{2x} = \int e^{5x}$$

$$ye^{2x} = \frac{e^{5x}}{5} + c$$

$$y = \frac{e^{3u}}{5} + \frac{c}{e^{2x}}$$

Question 4

$$dy/du + y/x = y^3$$

$$z = y^{1-n} \quad z = y^{1-3} = y^{-2}$$

$$dz/du = (1-n)y^{-n}\, \partial y/dx = 1 - 3y^{-3}\, dy/dx = -2y^{-3}\, \partial y/du$$

$$\frac{dy}{dx} + \frac{y}{x} = y^3$$

Dividing through by y^3

$$y^{-3}\, dy/dx + 1/xy^2 = 1$$

multiply by -2

$$-2y^{-3}\, dy/dx - 2/xy^2 = -2$$

$$-2y^{-3}\, dy/dx - 2/x\, y^2 = -2$$

$$dz/dx - 2/x\, z = -2$$

$$IF = e^{\int P\partial x} = e^{-2/x\, \partial x}$$

$$= e^{-2\ln x} = x^{-2}$$

$$x^{-2} \cdot z = \int -2 \cdot x^{-2}$$

$$zx^{-2} = \frac{-2x^{-2+1}}{-2+1}$$

$$zx^{-2} = 2x^{-1} + c$$

Recall $z = y^{-2}$

$$y^{-2}x^{-2} = 2x^{-1} + c$$

$$y^2 = \frac{x^{-2}}{2x^{-1}} + c$$

$$y^2[2x^{-1} + c] = x^{-2}$$

Question 3

$$x\, \partial y/dx = x^2 + 2x - 3$$

Using direct integration

$$dy/du = x^2/x + \frac{2x}{x} - 3/x$$

$$dy/du = x + 2 - 3/x$$

$$\int dy = \int \left(x + 2 - 3/x\right) \partial x$$

$$y = \int (x + 2 - 3/x)\, dx$$

$$y = \frac{x^2}{2} + 2x - 3\ln x + c$$

$$y^2x^2[2x^{-1} + c] = 1$$

$$y^2x^2 2x^{-1} + y^2x^2c = 1$$

$$y^2x\, [cx + 2] = 1$$

Question 5

$x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$

$\frac{dy}{dx} = \frac{x^3 \sin 3x}{x^2} + \frac{4}{x^2}$

$\frac{dy}{dx} = x \sin 3x + \frac{4}{x^2}$

$\int \frac{dy}{dx} = \int (x \sin 3x + \frac{4}{x^2}) \partial x$

$y = \int x \sin 3x + \int \frac{4}{x^2}$

Integrate $\int x \sin 3x$ using integration by parts

$\int x \sin 3x \qquad \int u \partial v = uv - \int v \partial u$

$u = x \qquad du = 1$

$dv = \sin 3x \qquad v = \frac{-\cos 3x}{3}$

$-\frac{x \cos 3x}{3} - \int -\frac{\cos 3x}{3} \cdot 1$

$-\frac{x \cos 3x}{3} + \frac{\sin 3x}{9}$

$y = \frac{-x \cos 3x}{3} + \frac{\sin 3x}{9} + \int \frac{4}{x^2}$

$y = -\frac{x \cos 3x}{3} + \frac{\sin 3x}{9} + \frac{4x^{-2+1}}{-2+1}$

$y = -\frac{x \cos 3x}{3} + \frac{\sin 3x}{9} - \frac{4}{x} + c$

Question 6

$(x^3 + xy^2) \frac{dy}{dx} = 2y^3$

$y = vx$

$\frac{dy}{dx} = v + u \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{2y^3}{x^3 + xy^2}$

$\frac{dy}{dx} = \frac{2(vx)^3}{x^3 + x(vx)^2}$

$\frac{dy}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$

$\frac{dy}{dx} = \frac{2v^3}{1+v^2}$

$v + x\frac{dv}{dx} = \frac{2v^3}{1+v^2}$

$x \frac{dv}{dx} = \frac{2v^3}{1+v^2} - v$

$x \frac{dv}{dx} = \frac{2v^3 - v(1+v^2)}{1+v^2}$

$x \frac{dv}{dx} = \frac{2v^3 - v - v^3}{1+v^2}$

$x \frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$

$\int \frac{1+v^2}{v^3 - v} dv = \int \frac{1}{x} dx$

Integrating using partial fraction

$\frac{1+v^2}{v^3 - v} = \frac{1+v^2}{v(v^2-1)} = \frac{1+v^2}{(v)(v+1)(v-1)}$

$\frac{1+v^2}{(v)(v+1)(v-1)} = \frac{A}{v} + \frac{B}{v+1} + \frac{C}{v-1}$

$1+v^2 = A(v+1)(v-1) + B(v)(v-1) + C(v)(v+1)$

let $v = 1$

$2 = A(0) + B(0) + C(1)(2)$

$2 = 2C$

$C = 1$

let $v = -1$

$2 = A(0) + B(-1)(-2) + C(0)$

$B = 1$

let $v = 0$

$1 = A(1)(-1) + B(0) + C(0)$

$A = -1$

$\int \frac{-1}{v} + \int \frac{1}{v+1} + \int \frac{1}{v-1}$

$-\ln v + \ln(v+1) + \ln(v-1)$

$-\ln v + \ln(v+1) + \ln(v-1) = \int \frac{1}{x} dx$

$-\ln v + \ln(v+1) + \ln(v-1) = \ln x + c$

let $c = \ln A$

$\ln\left[\frac{(v+1)(v-1)}{v}\right] = \ln x + \ln A$

$$\frac{(v+1)(v-1)}{v} = Ax$$

$$y = vx$$

$$v = \frac{y}{x}$$

$$\frac{\left(\frac{y}{x}+1\right)\left(\frac{y}{x}-1\right)}{y/x} = Ax$$

$$\frac{\frac{y^2}{x^2}-1}{y/x} = Ax$$