JOHN-UGWU NNEOMA FAVOUR
18/ENG02/052
COMPUTER ENGINEERING.

6) $\frac{dy}{dx} = 2\sinh x - y\tanh x$

$\frac{dy}{dx} + y\tanh x = \sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P\,dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} \cdot dx$

$\cosh x = u$

$\int \frac{\sinh x}{u} \cdot dx$

$u = \cosh x$

$\frac{du}{dx} = \sinh x$

$dx = \frac{du}{\sinh x}$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x}$

$= \int 1/u \cdot du$

$= \ln u$

$= \ln \cosh x$

$= e^{\int P \cdot dx}$

$= e^{\ln \cosh x} = \cosh x$

$y \cdot IF = \int Q \cdot IF \cdot dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x \cdot dx$

$2\sinh x \cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x \cdot dx$

$y \cdot \cosh x = 1/2 \times 2\cosh 2x + c$

$y \cdot \cosh x = \cosh 2x + c$

$y = \frac{\cosh 2x + c}{\cosh x}$

b. $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2$

$\int P \cdot dx = 2x$

$Q = e^{3x}$

$IF = e^{\int P \cdot dx}$

$= e^{2x}$

$y \cdot IF = \int Q \cdot IF \cdot dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} dx$

$y \cdot e^{2x} = \int e^{5x} dx$

$y \cdot e^{2x} = 1/5 e^{5x} + c$

$y = \frac{1/5 e^{5x} + c}{e^{2x}}$

c. $x \, dy/dx = x^2 + 2x - 3$

$dy/dx = x + 2 - \frac{3}{x}$

$\int \frac{dy}{dx} = \int x + 2 - \frac{3}{x} dx$

$y = \frac{x^2}{2} + 2x - 3 \ln x + c$

d. $\frac{dy}{dx} + y/x = y^3$

$dy/dx \, y^{-3} + y^{-2}/x = 1 \quad \ldots (i)$

$z = y^{1-n}$

$z = y^{1-3}$

$z = y^{-2} \quad \ldots (ii)$

$\frac{dz}{dy} = -2y^{-3} \, dy/dx \quad \ldots (iii)$

Multiply eqn by (i-ii)

$-2y^{-3} dy/dx - 2y^{-3}/x = -2$

$\frac{dz}{dy} = -2y^{-3} \frac{dy}{dx}$

sub eqn (i) into (4)

$\frac{dz}{dy} - \frac{2z}{x} = -2$

$\therefore P = -2/x \quad Q = -2$

$\int P \cdot dx = -2 \ln x$

$IF = e^{-2\ln x}$

$= x^{-2}$

$z \cdot IF = \int Q \cdot IF \, dx$

$z x^{-2} = \int -2x^{-2} \, dx$

$z x^{-2} = \frac{-2x^{-1}}{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$y^{-2} = x(2 + cx)$

$z = y^{-2}$

$y^{-2} = x(x + cx)$

$\frac{1}{y^2} = x(2 + cx)$

$y^2 = \frac{1}{2}(2 + cx)$

e. $x^2 \, dy/dx = x^3 \sin 3x + 4$

$dy/dx = x \sin 3x + \frac{4}{x^2}$

$\int dy/dx = \int x \sin 3x + \int 4x^{-2}$

$= \frac{1}{3} \cos 3x + \int \frac{1}{3} \cos 2x + 4x^{-1}/_{-1}$

$= \frac{-x \cos 3x}{3} + \frac{\sin 3x}{4} - 4x^{-1}$

$= \frac{\sin 3x}{4} - \frac{x \cos 3x}{3} - \frac{4}{x}$

(f) $(x^3 + xy^2) \, dy/dx = 2y^3$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dy}{dx}$

$v + x \, dy/dx = \frac{2(vx)^3}{x^3 + v^2 x^3}$

$v + x \, dy/dx = \frac{x^3}{x^3}\left(\frac{2v^3}{(1+v^2)}\right.$

$= \frac{2v^3 - v(1+v^2)}{1+v^2}$

$$= \frac{2v^3 - v - v^3}{1+v^2}$$

$$x \cdot dv = v^3$$

$$\frac{1+v^2}{v^3-u} \cdot du = \frac{1}{x} \cdot dx$$

$$v(v-1)(v+1) = v^3 - u$$

$$\frac{1+v^2}{v^3-v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$$

$$1+v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$$

$$1+1^2 = B(1)(2)$$

$$2 = 2B \quad ; \quad B = 1$$

$$v = -1$$

$$1+(-1)^2 = C(-1)(-1-1)$$

$$2 = 2C \quad ; \quad C = 1$$

$$v = 0$$

$$\therefore \quad 1+(0)^2 = A(0-1)(0+1)$$

$$1 = A(-1)(1)$$

$$1 = -1A \quad ; \quad A = -1.$$

$$\int\left(-\frac{1}{v} + \frac{1}{v-1} + \frac{1}{v+1}\right) du = \int \frac{1}{x} \cdot dx$$

$$-\ln v + \ln(v-1) + \ln(v+1) = \ln x$$

$$\ln(v-1)(v+1) - \ln v = \ln x$$

$$\frac{v^2-1}{v} = \ln x$$

$$y = vx$$

$$v = y/x \quad ; \quad (y/x)^2 - 1 = Ax$$

$$y^2/x^2 - 1 = Ax \cdot \frac{y}{x}$$

$$y^2/x^2 - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay.$$

$$y^2 - x^2 = Ayx^2$$

$$y^2 = Ax^2 y + x^2$$

$$y^2 = x^2(Ay + 1)$$