Assignment

a) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

$P = \tanh x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x = u$

$\int \frac{\sinh x}{u} dx$

$u = \cosh x$

$\frac{du}{dx} = \sinh x$

$\therefore dx = \frac{du}{\sinh x}$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x}$

$\int \frac{1}{u} du = \ln u = \ln \cosh x$

$If = e^{\ln \cosh x}$

$= e^{\ln \cosh x}$

$If = \cosh x$

Then $y \cdot If = \int Q \cdot If \, dx$

$y \cdot \cosh x = \int -\sinh x \cdot \cosh 2x dx$

and $2\sinh x \cosh x = \sinh(2x)$

$\therefore 2\sinh x \cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x \, dx$

$y \cdot \cosh x = \frac{1}{2} \cdot \cosh 2x + c$

$\cosh x y = \frac{\cosh 2x}{2} + c$

$y = \frac{\cosh 2x}{2} + c$

$\cosh x$

$y = \frac{\cosh 2x + c}{\cosh x}$

Let $If = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2 \quad \int P dx = 2x$

$Q = e^{3x}$

$If = e^{\int P dx} = e^{2x}$

$\therefore y \cdot If = \int Q \cdot If \, dx$

$y \cdot e^{3x} = \int e^{3x} \cdot e^{2x} dx$

$y \cdot e^{2x} = \int e^{5x} dx$

$y \cdot e^{2x} = \frac{1}{5} e^{5x} + c$

$y = \frac{1}{5} e^{5x} + ce^{-2x}$

c) $\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 \cdot \frac{5}{x}$

$\int \frac{dy}{dx} dx = \int x + 2 \cdot \frac{5}{x} dx$

$y = \frac{x^5}{2} + 2x - 3\ln x + c$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$\frac{dy}{dx} y^{-3} + \frac{y^{-4}}{x} = 1 \quad ---①$

$z = y^{1-n}$

$n = 3$

$z = y^{-2} \quad ---②$

$\frac{dz}{dy} = -2y^{-3} \frac{dy}{dx} \quad ---③$

multiply equ① by equ②

$-2y^{-3} \frac{dy}{dx} - \frac{2y^{-2}}{x} = -2 \quad ---④$

$\frac{dz}{dy} = \frac{dz}{dy} = \frac{-2y^{-3}}{\frac{dx}{dy}}$

a) Substitute equ ③ and equ ⑤ into ④.

$$\frac{dz}{dy} - \frac{2z}{x} = -2$$

$$\therefore P = -\frac{2}{x}$$

$$Q = -2$$

$$\int P\,dx = -2\ln x$$

$$IF = e^{-2\ln x} = x^{-2}$$

$$z \cdot IF = \int Q \cdot IF\,dx$$

$$z \cdot x^{-2} = \int -2x^{-2}\,dx$$

$$= -2\int x^{-2}\,dx$$

$$= \frac{-2x^{-1}}{-1} + c$$

$$z \cdot x^{-2} = 2x^{-1} + c$$

$$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$$

$$z = 2x + cx^{2}$$

$$z = y^{-2}$$

$$y^{-2} = x(2 + cx)$$

e) $x^2 \frac{dy}{dx} = x^3 \sin^3 x + 4$

$$\frac{dy}{dx} = x\sin 3x + \frac{4}{x^2}$$

$$\int \frac{dy}{dx} = \int x\sin 3x + \int 4x^{-2}$$

$$= \frac{1}{3}\cos 3x + \sin 3x / -4x^{-1}$$

f) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dy}{dx}$$

$$v + x\frac{dv}{dx} = \frac{2(vx)^3}{x^3} + v^2x^3$$

$$v + x\frac{dv}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$$

$$x\frac{dv}{dx} = \frac{2v}{1+v} - v$$

$$= \frac{2v^3 - v(1+v^2)}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$$