

Question 1

$$\frac{dy}{dx} + y \cosh x = 2 \sinh x$$

Solving using Integrating factor

$$I.F = e^{\int P dx}$$

$$I.F = e^{\int \cosh x dx}$$

$$I.F = e^{\int \frac{e^{\sinh x} dx}{\cosh x}} = e^{\ln(\cosh x)}$$

$$I.F = \cosh x$$

$$y \cdot I.F = \int Q \cdot I.F dx$$

$$y \cdot \cosh x = \int 2 \sinh x \cdot \cosh x$$

Integrating R.H.S

$$\int 2 \sinh x \cosh x$$

$$\text{let } u = \cosh x$$

$$\frac{du}{dx} = \sinh x \quad dx = \frac{du}{\sinh x}$$

$$\int 2 \sinh x \cdot u \cdot \frac{du}{\sinh x}$$

$$\int 2u du$$

$$\int 2u du$$

$$= \frac{2u^2}{2}$$

$$u^2 = \cosh^2 x$$

$$y \cdot \cosh x = \cosh^2 x + C$$

$$y = \frac{\cosh^2 x}{\cosh x} + \frac{C}{\cosh x}$$

$$y = \cosh x + C \cosh^{-1} x$$

Question 3

$$dy/dx + y = e^{2x}$$

Solving using integrating factor

$$I.F. = e^{\int 1 dx} = e^x = e^{2x}$$

$$y \cdot I.F. = \int I.F. \cdot R.H.S. dx$$

$$y \cdot e^{2x} = \int e^{2x} \cdot e^{2x} dx$$

$$y \cdot e^{2x} = \int e^{4x} dx + C$$

$$y \cdot e^{2x} = \frac{e^{4x}}{4} + C$$

$$y = \frac{e^{2x}}{4} + \frac{C}{e^{2x}}$$

Question 4

$$2x^2 dy/dx = 2y^3 + 2x \Rightarrow$$

Solving using integration

$$\frac{dy}{y^3} = \frac{2x^2 + 2x}{2x^2} dx = \frac{2x^2}{2x^2} + \frac{2x}{2x^2} dx = 1 + \frac{1}{x} dx$$

$$\int \frac{dy}{y^3} = \int (1 + \frac{1}{x}) dx$$

$$y = \int (2 + 2 - \frac{2}{x}) dx$$

$$y = \int (4 + 2 - \frac{2}{x}) dx$$

$$y = \frac{2x^2}{2} + 2x - 2 \ln|x| + C$$

Question 4

$$dy/dx + y/x = y^3$$

$$Z = y^{1-n} = y^{1-3} = y^{-2}$$

$$dZ/dx = (1-n)y^{-n-1} dy/dx = 1 \cdot (-2)y^{-3} dy/dx = -2y^{-3} dy/dx$$

$$\frac{dy}{dx} + \frac{y}{x} = y^3$$

$$-2y^{-3} dy/dx + \frac{y}{x} = y^3$$

$$-2y^{-3} dy/dx + \frac{y}{x} = y^3$$

$$-2y^{-3} dy/dx - \frac{2y}{x} = -2$$

$$-2y^{-3} dy/dx - \frac{2y}{x} = -2$$

$$dZ/dx - \frac{2}{x} Z = -2$$

$$I.F. = e^{\int -2/x dx} = e^{-2 \ln|x|} = x^{-2}$$

$$Z \cdot x^{-2} = \int -2 \cdot x^{-2} \cdot x^{-2} dx$$

$$Z \cdot x^{-2} = \frac{-2x^{-4}}{-4} + C$$

$$Z \cdot x^{-2} = \frac{-2x^{-4}}{-4} + C$$

$$Z \cdot x^{-2} = \frac{-2x^{-4}}{-4} + C$$

$$Z \cdot x^{-2} = \frac{-2x^{-4}}{-4} + C$$

$$Z \cdot x^{-2} = \frac{-2x^{-4}}{-4} + C$$

$$Z \cdot x^{-2} = \frac{-2x^{-4}}{-4} + C$$

$$Z \cdot x^{-2} = \frac{-2x^{-4}}{-4} + C$$

$$Z \cdot x^{-2} = \frac{-2x^{-4}}{-4} + C$$

$$Z \cdot x^{-2} = \frac{-2x^{-4}}{-4} + C$$

$$Z \cdot x^{-2} = \frac{-2x^{-4}}{-4} + C$$

$$Z \cdot x^{-2} = \frac{-2x^{-4}}{-4} + C$$

$$Z \cdot x^{-2} = \frac{-2x^{-4}}{-4} + C$$

$$y^2 x^2 [2x^2 + 2] = 1$$

$$y^2 x^2 [2x^2 + 2] = 1$$

$$y^2 x^2 [2x^2 + 2] = 1$$

Question 5

$$x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$$

$$\frac{dy}{dx} = \frac{x^3 \sin 3x + 4}{x^2}$$

$$\frac{dy}{dx} = x \sin 3x + \frac{4}{x^2}$$

$$\int \frac{dy}{dx} = \int (x \sin 3x + \frac{4}{x^2}) dx$$

$$y = \int x \sin 3x + \int \frac{4}{x^2}$$

Integrate $\int x \sin 3x$ using integration by parts
 $\int u dv = uv - \int v du$

$$u = x \quad du = 1$$

$$dv = \sin 3x \quad v = -\frac{\cos 3x}{3}$$

$$-\frac{x \cos 3x}{3} - \int -\frac{\cos 3x}{3} \cdot 1$$

$$-\frac{x \cos 3x}{3} + \frac{\sin 3x}{9}$$

$$y = -\frac{x \cos 3x}{3} + \frac{\sin 3x}{9} + \int \frac{4}{x^2}$$

$$y = -\frac{x \cos 3x}{3} + \frac{\sin 3x}{9} + \frac{4x^{-2+1}}{-2+1}$$

$$y = -\frac{x \cos 3x}{3} + \frac{\sin 3x}{9} - \frac{4}{x} + C$$

Question 6

$$(x^3 + xy^2) \frac{dy}{dx} = 2y^3$$

$$y = vx$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{2y^3}{x^3 + xy^2}$$

$$\frac{dy}{dx} = \frac{2(vx)^3}{x^3 + x(vx)^2}$$

$$\frac{dy}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$$

$$\frac{dy}{dx} = \frac{2v^3}{1+v^2}$$

$$v + x \frac{dv}{dx} = \frac{2v^3}{1+v^2}$$

$$x \frac{dv}{dx} = \frac{2v^3}{1+v^2} - v$$

$$x \frac{dv}{dx} = \frac{2v^3 - v(1+v^2)}{1+v^2}$$

$$x \frac{dv}{dx} = \frac{2v^3 - v - v^3}{1+v^2}$$

$$x \frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$$

$$\int \frac{1+v^2}{v^3 - v} dv = \int \frac{1}{x} dx$$

Integrating using partial fraction

$$\frac{1+v^2}{v^3 - v} = \frac{1+v^2}{v(v^2-1)} = \frac{1+v^2}{(v)(v+1)(v-1)}$$

$$\frac{1+v^2}{(v)(v+1)(v-1)} = \frac{A}{v} + \frac{B}{v+1} + \frac{C}{v-1}$$

$$1+v^2 = A(v+1)(v-1) + B(v)(v-1) + C(v)(v+1)$$

$$\text{let } v = 1$$

$$2 = A(2) + B(0) + C(1)(2)$$

$$2 = 2C$$

$$C = 1$$

$$\text{let } v = -1$$

$$2 = A(0) + B(-1)(-2) + C(0)$$

$$B = 1$$

$$\text{let } v = 0$$

$$1 = A(1)(-1) + B(0) + C(0)$$

$$A = -1$$

$$\int \frac{-1}{v} + \int \frac{1}{v+1} + \int \frac{1}{v-1}$$

$$-\ln v + \ln(v+1) + \ln(v-1)$$

$$-\ln v + \ln(v+1) + \ln(v-1) = \int \frac{1}{x} dx$$

$$-\ln v + \ln(v+1) + \ln(v-1) = \ln x + C$$

$$\text{let } C = \ln A$$

$$\ln \left[\frac{(v+1)(v-1)}{v} \right] = \ln x + \ln A$$

$$(W+D)(y-\dots) = Ax$$

$$y = \frac{W}{V}x$$

$$V = \frac{W}{x}$$

$$\left(\frac{W}{x} + \dots\right) \left(\frac{W}{x} - \dots\right) = Ax$$

$$\frac{\frac{W}{x} + \dots}{\frac{W}{x}} - 1 = Ax$$