[illegible]

18/ENG 06/035

18th March 2020

a $\frac{\delta y}{\delta x} = 2\sinh x - y\tanh x$

$\frac{\delta y}{\delta x} + y\tanh x = 2\sinh x$

$P = \tan x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x = u \quad \frac{du}{dx} = \sinh x$

$dx = du/\sinh x$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x} = \int \frac{1}{u} du = \ln u$

$= \ln \cosh x$

$I_F = e^{\int P dx} = e^{\ln \cosh x} \quad \therefore I_F = \cosh x$

$y \cdot I_F = \int Q \, I_F \, dx$

$y = \cosh x = \int \sin 2x dx$

$y \cdot \cosh x = \frac{1}{\cancel{2}} \cdot \cancel{2} \cosh 2x + C$

$\cosh x y = \cosh 2x + C$

$y = \frac{\frac{\cosh 2x}{2} + C}{\cosh x}$

$y = \frac{\cosh 2x + 2C}{\cosh x}$

let $2C = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b $\frac{\delta y}{\delta x} + 2y = e^{3x}$

$P = 2 \quad \int P dx = 2x$

$Q = e^{3x}$

$I_F = e^{\int P dx} = e^{2x}$

$y \cdot I_F = \int Q \cdot I_F \, dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} dx$

$y \cdot e^{2x} = 1/5 e^{5x} + c$

$y = \frac{\frac{1}{5}e^{5x} + c}{e^{3x}}$

c $x \frac{dy}{\delta x} = x^2 + 2x - 3$

$\frac{dy}{\delta x} = x + 2 - 3/x$

$\int \frac{dy}{\delta x} = 5x + 2 \; 3/x \quad dx$

$y = x \frac{x^2}{2} + 2x = 3 \ln x + C$

d $\frac{\delta y}{\delta x} + \frac{y}{x} = y^3$

$\frac{dx}{\delta x} y^{-3} + y^{-3}/x = 1 \quad \ldots (1)$

$z = y^{1-n} \qquad n = 3$

$z = y^{1-3} = z = y^{-2} \quad \ldots (2)$

$\frac{\delta x}{\delta y} = 2y^{-3} \frac{dy}{\delta x} \quad \ldots (3)$

multiplying (eqns) and $y(1-n)$

$-2y^{-3} \frac{dy}{\delta x} - 2y \frac{z}{x} = -7$

and $\delta x/\delta y = -2y^{-3} \frac{dy}{\delta x}$

sub eqn 2 and 3 onto (1)

$\frac{\delta x}{\delta y} - 2x/x = -2$

$\therefore P = -2/x \qquad Q = -2$

$\int p\,\delta x = -2\ln x$

$I_f\ e^{-2\ln x} = x^{-2}$

$z \cdot I_f = \int Q \cdot I_f \cdot \delta x$

$z \cdot x^{-2} = \int -2x^{-2}\,\delta x$

$= -\frac{2x^{-1}}{1} + c$

$2x^{-2} = 2x^{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$y^{-2} = x(2 + cx)$

$\frac{1}{y^2} = x\,[2 + cx]$

$y^2 = \frac{1}{x}(2 + cx))$

$y = \sqrt{\frac{1}{x}(2 + cx)}$

e) $x^2 \frac{\delta y}{\delta x} = x^3 \sin 3x + 4$

$\frac{dy}{\delta x} = x \sin 3x + 4$

$\int \frac{\delta y}{\delta x} = \int x^4 \sin 3x + \int 4x^{-2}$

$= \frac{1}{3} \cos 3x + \int \frac{1}{3} \cos 3x + 6x^{-1}$

$= -\frac{x \cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1}$

$y = \frac{\sin 3x}{4} - \frac{x \cos 3x}{3} - \frac{4}{x}$

f $(x^3 + xy^2) \frac{dy}{\delta x} = 2y^3$

$y = vx$

$\frac{dy}{\delta x} = v + x \frac{dv}{\delta x}$

f) $\cancel{x^3 y + x}\ \frac{\delta y}{\delta x} = \frac{2y^3}{x^3 + xy^2}$

$= v + x\frac{\delta v}{\delta x} = \frac{2(vx)^3}{x^3 + x(vx)^2)}$

$v + x\frac{\delta v}{\delta x} = \frac{2(vx)^3}{x^3 + x(vx)^2)}$

$v + x\frac{\delta v}{\delta x} = \frac{2(vx)^3}{(x^3 + x(vx)^2}$

$v + x\frac{dv}{\delta x} = \frac{2v^2x^3}{x^3 + v^2x^3}$

$v + x\frac{dv}{\delta x} = \frac{2x^3(2v^3)}{x^3(1+v^2)}$

$v + x\frac{dv}{\delta x} = \frac{2v^3}{1+v^2}$

$\frac{1+v^2\delta v}{v^3 - v} = \frac{1}{x}\delta x$

$\int \frac{1+v^2}{v^3-v} = \int \frac{1}{x}\delta x$

$= \ln v + \ln(v-1) + \ln(v+1)\quad \ln x + c$

$= \ln\left(\frac{y}{x}\right) + \ln\left(\frac{y}{x} + 1\right) + \ln\left(\frac{y}{x} + 1\right)$

$= \ln x + c$