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18/ENG08/020
BIOMEDICAL ENGINEERING

1.] $\frac{dy}{dx} + y\tanh x = \sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int p \cdot dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} \cdot dx$

$\cosh x = u$

$\int \frac{\sinh x}{u} = dx$

$u = \cosh x$

$\frac{dy}{dx} = \sinh x$

$dx = \frac{du}{\sinh x}$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x}$

$= \int \frac{1}{u} \cdot dx$

$= \ln u$

$= \ln \cosh x$

$If = e^{\int p \cdot dx}$

$= e^{\ln \cosh x}$

$= \cosh x$

$y \cdot If = \int Q \cdot If \cdot dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x \cdot dx$

$2\sinh x \cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x \cdot dx$

$y \cdot \cosh x = \frac{1}{2} \times 2\cos 2x + C$

$y \cdot \cosh xy = \cosh 2x + C$

$y = \frac{\cosh 2x}{\cosh x} + C$

2.] $\frac{dy}{dx} + 2y = e^{5x}$

$P = 2$

$\int P dx = 2x$

$Q = e^{5x}$

$IF = e^{\int P dx}$

$= e^{2x}$

$y \cdot IF = \int Q \cdot IF \cdot dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} \cdot dx$

$y \cdot e^{2x} = \int e^{5x} \cdot dx$

$y \cdot e^{2x} = \frac{1}{5} e^{5x} + C$

$y = \frac{\frac{1}{5}e^{5x} + c}{e^{2x}}$

3.] $x \frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\int \frac{dy}{dx} = \int x + 2 - \frac{3}{x} \, dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

4.] $\frac{dy}{dx} + \frac{y}{x} = y^3$

$\frac{dy}{dx} y^{-3} + \frac{y^{-2}}{x} = 1$ —— (i)

$z = y^{1-n}$

$z = y^{1-3}$

$z = y^{-2}$ —— (ii)

$\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$ —— (iii)

Multiply eqn by $(1-n)$

$-2y^{-3}\frac{dy}{dx} - \frac{2y^{-2}}{x} = -2$

$\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$

Sub eqn (2) into (4)

$\frac{dz}{dy} - \frac{2z}{x} = -2$

$\therefore P = \frac{-2}{x}; \; Q = -2$

$\int P \cdot dx = -2\ln x$

$If = e^{-2\ln x}$

$= -2$

$z \cdot IF = \int Q \cdot If \cdot dx$

$2x^{-2} = \int -2x^{-2} dx$

$2x^{-2} = \frac{-2x^{-1}}{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$y^{-2} = x(x + cx)$

$y^{-2} = x(2 + cx)$

$y^2 = \frac{1}{2(2+cx)}$

5.] $x^2 \frac{dy}{dx} = x^3\sin 3x + 4$

$\frac{dy}{dx} = x^3\sin 3x + \frac{4}{x^2}$

$\int \frac{dy}{dx} = \int \left(x\sin 3x + \frac{4}{x^2}\right)$

$y = uv - \int v du$

$-\frac{x\cos 3x}{3} - \int \frac{\cos 3x}{3}$

$\frac{-x\cos 3x}{3} - \frac{1}{3}\int \cos 3x$

$\frac{-x\cos 3x}{3} - \frac{1}{3}\frac{\sin 3x}{3}$

$-\frac{x\cos 3x}{3} - \frac{\sin 3x}{9} - (-4x^{-1}) + C$

$y = -\frac{x\cos 3x}{3} - \frac{\sin 3x}{9} - \frac{4}{x} + C$

$c = 1$

$v = 0$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = -1A$

$A = -1$

$\int \left(\frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right) du$

$- \int \frac{1}{4} \cdot dx$

$$-\ln v + \ln(v-1) + \ln(v+1) = \ln x$$

$$\ln(v-1)(v+1) - \ln v = \ln x$$

$$\frac{v^2-1}{v} = \ln x$$

$$y = vx$$

$$v = y/x$$

$$(y/x)^2 - 1 = Ax$$

$$\frac{y^2}{x^2} - 1 = Ax \cdot \frac{y}{x}$$

$$\frac{y^2}{x^2} - 1 = Ay$$

$$\frac{y^2 - x^2}{x^2} = Ay$$

$$y^2 - x^2 = Ayx^2$$

$$y^2 = Ax^2y + x^2$$

$$y^2 = x^2(\cancel{A}y + 1)$$

6.] $(x^2+xy^2)\frac{dy}{dx} = 2y^3$

$$\frac{dy}{dx} = \frac{2y^3}{(x^2+xy^2)}$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{2y^3}{(x^2+xy^2)}$$

$$v + x\frac{dv}{dx} = \frac{2(vx)^3}{x^2+x(vx)^2}$$

$$v + x\frac{dv}{dx} = \frac{2v^3x^3}{x^2+v^2x^3}$$

$$v + x\frac{dv}{dx} = \frac{2v^3x^3}{x^2(1+v^2)}$$

$$v + x\frac{dv}{dx} = \frac{2v^3}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{2v^3}{1+v^2} - \frac{v}{1}$$

$$x\frac{dv}{dx} = \frac{2v^2 - v(1+v^2)}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{2v^2 - v - v^3}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{2v^3 - v^3 - v}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{v^3 - v}{1+v^2}.$$

$$x - du = v^3$$

$$\frac{1+v^2}{v^3-u} \cdot du = \frac{1}{x} \cdot dx$$

$$x\frac{dv}{dx} = \frac{v(v^2+1)}{1+v^2} \Big/ \frac{(v^3-v)}{(1+v^2)x}$$

$-v^2)x\, dv$ | $v(v-1)(v+1) = v^3 - v$

$)\, dv$ | $\dfrac{1+v^2}{v^3-v} = \dfrac{A}{v} + \dfrac{B}{v-1} + \dfrac{C}{v+1}$

v

$v^2 - v$ | $1 - v^2 = A(v-1)(v+1) + B(v)(v+1) + \cancel{C(v)}$

$v^2 - 1$ | $C(v)(v-1)$

$1 + 1^2 = B(1)(2)$

$2 = 2B$

$B = 1$

$v = -1$

$1 + (-1)^2 = C(-1)(-1-1)$

$2 = 2C$