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18/ENG02/025
Computer Engineering
ENG 282
Assignment

(1) Solve.

a) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

$P = \tanh x, \quad Q = 2\sinh x$

$\int P\,dx = \int \tanh x = \int \frac{\sinh x}{\cosh x}\,dx$

where $\cosh x = u$

$\int \frac{\sinh x}{u}\,dx$

$u = \cosh x$

$du/dx = \sinh x, \quad dx = \frac{du}{\sinh x}$

$dx = \int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x}$

$\int \frac{1}{u}\,du = \ln u$

$= \ln \cosh x$

$IF = e^{\ln \cosh x}$

$IF = \cosh x$

Then $y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x\,dx$

$2\sin x \cos x = \sin(2x)$

$2\sinh x \cosh x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x\,dx$

$y \cdot \cosh = \frac{1}{2}\cosh 2x + c$

$\cosh + y = \frac{\cosh 2x}{2} + c$

$y = \frac{\frac{\cosh 2x}{2} + c}{\cosh x}$

$y = \frac{\cosh 2x + 2c}{\cosh x}$

let $2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$

soln.

$P = 2 \quad \int P\,dx = 2x$

$Q = e^{3x}$

$IF = e^{\int P\,dx} = e^{2x}$

$y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}\,dx$

$y \cdot e^{2x} = \int e^{5x}\,dx$

$y \cdot e^{2x} = \frac{1}{5}e^{5x} + c$

$y = \frac{\frac{1}{5}e^{5x} + c}{e^{2x}}$

$y = \frac{1}{5}e^{3x} + Ce^{-2x}$

c) $x\frac{dy}{dx} = x^2 + 2x - 3$

soln

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\int \frac{dy}{dx}\,dx = \int x + 2 - \frac{3}{x}\,dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

d) $\frac{dy}{dx} + \frac{y}{x} = y^3$

soln

$\frac{dy}{dx}y^{-3} + \frac{y^{-3}}{x} = 1$ — (1)

$z = y^{1-n}$, $n = 3$

$z = y^{1-3}$, $z = y^{-2}$ — (2)

$\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$ — (3)

Multiply eqn (1) by 1-n.

$-2y^{-3}\frac{dy}{dx} - \frac{2y^{-2}}{x} = -2$ — (4)

$\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$

Substitute eqn 2 and 3 into eqn (4)

$\frac{dz}{dy} - \frac{2z}{x} = -2$

$P = -2/x \quad Q = -2$

$\int P\,dx = -2\ln x$

$IF = e^{-2\ln x} = x^{-2}$

$z \cdot IF = \int Q \cdot IF\,dx$

$z \cdot x^{-2} = \int -2x^{-2}\,dx$

$= -2\int x^{-2}\,dx$

$= \frac{+2x^{-1}}{+1} + c$

$z \cdot x^{-2} = 2x^{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$y^{-2} = x(2 + cx)$

$1/y^2 = x(2 + cx)$

$y^2 = \frac{1}{x(2 + cx)}$

$\therefore y = \sqrt{\frac{1}{x(2+cx)}}$

$y = \frac{1}{\sqrt{x(2+cx)}}$

e) $x^2\frac{dy}{dx} = x^2\sin^3 x + 4$

soln

$\frac{dy}{dx} = x\sin 3x + \frac{4}{x^2}$

$\int \frac{dy}{dx} = \int x^4 \sin 3x + \int 4x^{-2}$

$= \frac{1}{3}\cos 3x + \int \frac{1}{3}\cos 3x + \frac{4x^{-1}}{-1}$

$= \frac{-2\cos 3x}{3} + \frac{\sin 3x}{9} + 4x^{-1}$

$= \frac{-2\cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1}$

$y = \frac{\sin 3x}{9} - \frac{x\cos 3x}{2} - \frac{4}{x}$

(B) $(x^3 + xy^2)\,{dy}/{dx} = 2y^3$

$y = vx$

$\frac{dy}{dx} = v + x\,{dv}/{dx}$

$v + x\frac{dv}{dx} = \frac{2(vx)^3}{x^2 + v^2x^3}$

$v + x\frac{dv}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$

$x\frac{dv}{dx} = \frac{2v^3}{1+v^2} - v$

$= \frac{2v^3 - v(1+v^2)}{1+v^2}$

$= \frac{2v^3 - v - v^3}{1+v^2}$

$\frac{x\,dx}{dx} = \frac{2v^3 - v - v^3}{1+v^2}$

$x \cdot dx = v^3$

$\frac{1+v^2}{v^3 - v} \cdot dx = \frac{1}{x} \cdot dx$

$v(v-1)(v+1) = v^3 - v$

$\frac{1+v^2}{v^3 - v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$1+v^2 = A(v-1)(v+1) + B(v)(v+1) + C(v)(v-1)$

$1+1^2 = B(1)(2)$

$2 = 2B$

$B = 1$, $v = -1$

$1+(-1)^2 = 1(-1)(-1-1)$

$C = 1$, $v = 0$

$1+(0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = -A$

$\int \left(\frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1}\right) dv = \int \frac{1}{x}\,dx$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + \ln A$

$\frac{v^2 - 1}{v} = Ax$

$\frac{y^2}{x^2} - 1 = Ax \cdot \frac{y}{x}$

$\frac{y^2}{x^2} - 1 = Ay$

$\frac{y^2 - x^2}{x} = Ay$

$y^2 - x^2 = Ayx^2$

$y^2 = Ax^2y + x^2$

$y^2 = x^2(Ay + 1)$