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Mechanical engineering
ENG 281

1) Solve;

a) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

$P = \tanh x$

$Q = 2\sinh x$

$\int P\,dx = \int \tanh x = \int \frac{\sinh x}{\cosh x}\,dx$

$\cosh x = u$

$\int \frac{\sinh x}{u}\,dx$

$u = \cosh x$

$du/dx = \sinh x$

$dx = \frac{du}{\sinh x}$

$\int \frac{\sinh x}{u} \cdot \frac{du}{\sinh x}$

$\int \frac{1}{u}\,du = \ln u$

$= \ln \cosh x$

$IF = e^{\ln \cosh x}$

$IF = \cosh x$

Then $y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot \cosh x = \int 2\sinh x \cdot \cosh x\,dx$

$2\sin x\cos x = \sin(2x)$

$2\sinh x \cosh x = \sinh(2x)$

$y\cosh x = \int \sinh 2x\,dx$

$y\cosh x = \frac{1}{2}\cosh 2x + c$

$\cosh x\, y = \frac{\cosh 2x}{2} + c$

$y = \frac{\frac{\cosh 2x}{2} + c}{\cosh x}$

$y = \frac{\cosh 2x + 2c}{\cosh x}$

let $2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2 \quad \int P\,dx = 2x$

~~$P = 2$~~ $\quad Q = e^{3x}$

$IF = e^{\int P dx} = e^{2x}$

$y \cdot IF = \int Q \cdot IF\,dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}\,dx$

$y \cdot e^{2x} = \int e^{5x}\,dx$

$y \cdot e^{2x} = 1/5\, e^{5x} + C$

$y = \frac{1/5\, e^{5x} + C}{e^{2x}}$

$y = 1/5\, e^{3x} + Ce^{-2x}$

1) $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - 3/x$

$\int \frac{dy}{dx}\,dx = \int x + 2 - \frac{3}{x} + dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

2) $\frac{dy}{dx} + \frac{y}{x} = y^3$

$\frac{dy}{dx} \cdot y^{-3} + \frac{y^{-3}}{x} = 1$ — ①

$= y^{1-n}$

$n = 3$

$= y^{1-3}$ — ②

$= y^{2}$

$\frac{dz}{dy} = -2y^{-3}\frac{dy}{dx}$ — ③

Then multiply equ ① by $1-n$

$-2y^{-3}\frac{dy}{dx} - \frac{2y^{-2}}{x} = -2$ — ④

$\frac{dz}{dy} = 2y^{-3}\frac{dy}{dx}$

Sub equ 2 and 3 into ④

$\frac{dz}{dy} - \frac{2z}{x} = -2$

$p = -2/x$

$Q = -2$

$\int p\,dx = 2\ln x$

$If = e^{2\ln x} = x^{-2}$

$z \cdot If = \int Q \cdot If\,dx$

~~$z \cdot If = \int Q \cdot If\,dx$~~

$z \cdot x^{-2} = \int -2x^{-2}\,dx$

$= -2\int x^{-2}\,dx$

$= \frac{+2x^{-1}}{+1} + c$

$= z \cdot x^{-2} = 2x^{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-3}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$z = y^{-2}$

$y^{-2} = x(2 + cx)$

$1/y^2 = x(2 + cx)$

$y^2 = \frac{1}{x(2+cx)}$

$\therefore y = \sqrt{\frac{1}{x(2+cx)}}$

$y = \frac{1}{\sqrt{x(2+cx)}}$

4) $x^2\frac{dy}{dx} = x^3\sin^3 x + 4$

$\frac{dy}{dx} = x\sin 3x + 4/x^2$

$\int \frac{dy}{dx} = \int x^4 \sin 3x + \int 4x^{-2}$

$= 1\cos 3x/3 + \int 1/3 \cos 3x + \frac{4x^{-1}}{-1}$

$$\frac{2\cos 3x}{3} + \frac{\sin 3x}{9} + \frac{4x^{-1}}{-1}$$

$$= -\frac{2\cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1}$$

$$y = \frac{\sin 3x}{9} - \frac{x\cos 3x}{2} - 4/x$$

(4) $(x^3 + xy^2)\,dy/dx = 2y^3$

$y = vx$

$$dy/dx = v + x\,dv/dx$$

$$v + x\,dv/dx = \frac{2(vx)^3}{x^2 + v^2 - x^2}$$

$$v + x\,dv/dx = \frac{x^3(2v^3)}{x^3(1+v^2)}$$

$$x\,dv/dx = \frac{2v^3}{1+v^2} - v$$

$$= \frac{2v^3 - v(1+v^2)}{1+v^2}$$

$$= \frac{2v^3 - v - v^2}{1+v^2}$$

$$x\,dv/dx = \frac{v^3 - v}{1+v^2}$$

(5) $(x^3 + xy^2)\,dy/dx = 2y^3$

$y = vx$

$dy/dx = v + x\,dv/dx$

$$v + x\,dv/dx = \frac{2(vx)^3}{x^2 + v^2x^2}$$

$\sigma = nq\mu_n$

$\sigma = nq(\mu_n + \mu_p)$

$\sigma = 2.5\times10^{13} \times 1.6\times10^{-19}\,(3800 + 1850)$

$\sigma = 2.26\times10^{-28}$

$$\text{Resistivity} = \frac{1}{2.26\times10^{-28}}$$

$= 4.424\times10^{27}$

$$v + x\,dv/dx = \frac{x^3(2v^3)}{x^3(1+v^2)}$$

$$x\,dv/dx = \frac{2v^3}{1+v^2} - v$$

$$= \frac{2v^3 - v(1+v^2)}{1+v^2}$$

$$= \frac{2v^3 - v - v^3}{1+v^2}$$

$$x\,dv/dx = \frac{2v^3 - v - v^3}{1+v^2}$$

$x \cdot dx = v^3$

$$\frac{1+v^2}{v^3 - v}\,dx = \frac{1}{x}\cdot dx$$

$v(v-1)(v+1) = v^3 - v$

$$\frac{1+v^2}{v^3 - v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$$

$1+v^2 = A(v-1)(v+1) + B(v)(1+v) + C(\ldots$

$1+1^2 = B(1)(2)$

$2 = 2B$

$B = 1$, $v = -1$

$1 + (-1)^2 = 1(-1)(-1-1)$

$e = 1, \quad v = 0$

$1 + (0)^2 = A(0-1)(0+1)$

$1 = A(-1)(1)$

$1 = -A$

$\int \cdot \left(-\frac{1}{v} + \frac{1}{v-1} + \frac{1}{v+1}\right) dv = \int \frac{1}{x} dx$

$-\ln v + \ln(v-1) + \ln(v+1) = \ln x + \ln A$

$\frac{v^2 - 1}{v} = Ax$

$\frac{y^2}{x^2} = 1 = Ax - \frac{y}{x}$

$\frac{y^2}{x^2} = 1 = Ay$

$\frac{y^2 - x^2}{x} = Ay$

$y^2 - x^2 = Ayx^2$

$y^2 = Ax^2y + x^2$

$y^2 = x^2(Ay+1)$