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18/ENG06/024
Mechanical Engineering

$dy/dx + y\tanh x = 2\sinh x$

$dy/dx + Py = Q$

$P = \tanh x \quad , \quad Q = 2\sinh x$

$\int p\,dx$

$= \int \tanh x \cdot dx$

$= \ln(\cosh x)$

$IF = e^{\ln \cosh x} = \cosh x$

$\cosh x \, dy/dx + y\cosh x \tanh x = 2\cosh x \sinh x$

$y \cdot IF = \int Q \cdot IF\,dx$

$y\cosh x = \int 2\sinh x \cosh x \, dx$

$y\cosh x = 2\int \sinh x \cosh x \cdot dx$

$y\cosh x = (\cosh^2 x) + C$

$y = \cosh x + \frac{C}{\cosh x}$

2. $dy/dx + 2y = e^{3x}$

$P = 2$

$Q = e^{3x}$

$IF \; e^{\int p\,dx}$

$\int p\,dx = \int 2\,dx$

$= 2x$

$IF = e^{2x}$

$y \cdot IF = \int Q \, IF\,dx$

$ye^{2x} = \int e^{3x} e^{2x}\,dx$

$ye^{2x} = \int e^{5x}\,dx$

$ye^{2x} = \frac{e^{5x}}{5} + \frac{C}{e^{2x}} \Rightarrow \frac{e^{3x}}{5} + \frac{C}{e^{2x}}$

$y = \frac{e^{5x}}{5e^{2x}} + \frac{C}{e^{2x}} = \frac{e^{3x}}{5} + \frac{C}{e^{2x}}$

3. $x\frac{dy}{dx} = x^2 + 2x - 3$ 5

$\frac{dy}{dx} = \frac{x^2}{x} + \frac{2x}{x} - \frac{3}{x}$

$\int dy = \int (x + 2 - \frac{3}{x}) dx$

$y = \int (x + 2 - \frac{3}{x}) dx$

$y = x^2/2 + 2x - 3\ln x + C$

4. $dy/dx + y/x = y^3$

$dy/dx + Py = Qy^n$ 6

$P = 1/x ; Q = 1, n = 3$

$y^{-3} dy/dx + \frac{y^{-2}}{x} = 1$

$z = y^{1-n} = y^{-2}$

$\frac{dz}{dx} = -2y^{-3} \frac{dy}{dx}$

$-2y^{-3} \frac{dy}{dx} + (\frac{-2y^{-2}}{x}) = -2$

$z . IF = \int Q\, IF\, dx$

$\int P dx = \int 1/x\, dx$

$\ln x$

$e^{\ln x} = x$

$Z x = \int x\, dx$

$2x = \frac{x^2}{2} + C$

$y^{-2} = \frac{x^2}{2x} + \frac{C}{x}$

$y^{-2} = x/2 + C/x$

$y = vx$

$\dfrac{v^2 - 1}{v} = Ax$

$\dfrac{y^2/x^2 - 1}{y/x} = Ax$ //

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$x^2\, dy/dx = x^3 \sin 3x + 4$

$dy/dx = x^3 \sin 3x + \frac{4}{x^2}$

$\int dy = \int (x \sin 3x) + \int 4/x^2$

$u = x \quad du = 1$

$dv = \sin 3x \quad v = \frac{-\cos 3x}{3}$

$\frac{-x\cos 3x}{3} - \int -\frac{\cos 3x}{3} \cdot 1$

$y = \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} + \frac{4x^{-2+1}}{-2+1}$

$y = \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} - \frac{4}{x} + c$

6. $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

$y = vx$

$\frac{dy}{dx} = v + x\, dv/dx$

$\frac{dy}{dx} = \frac{2y^3}{x^3 + xy^2}$

$\frac{dy}{dx} = \frac{2(vx)^3}{x^3 + x(vx)^2}$

$\frac{dy}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$

$\frac{dy}{dx} = \frac{x^3 \cdot (2v^3)}{x^5(1+v^2)}$

$\frac{dy}{dx} = \frac{2v^3}{1+v^2}$

$v + x\frac{dv}{dx} = \frac{2v^3}{1+v^2}$

$x\frac{dv}{dx} = \frac{2v^3 - v(1-v^2)}{1-v^2}$

$x\, dv/dx = \frac{2v^3 - v(1-v^2)}{1-v^2}$

$x\, dv/dx = \frac{2v^3 - v - v^3}{1+v^2}$

$x\frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$

$\int \frac{1+v^2}{v^3 - v}\, dv = \int 1/x\, dx$

$\frac{1+v^2}{v^3 - v} = \frac{1+v^2}{v(v^2-1)} = \frac{1+v^2}{v(v+1)(v-1)}$

$= \frac{A}{v} + \frac{B}{v+1} + \frac{C}{v-1}$

$1 + v^2 = A(v+1)(v-1) + B(v)(v-1) + C(v)(v+1)$

when $v = 1$, $C = 1$

when $v = -1$, $B = 1$

when $v = 0$, $A = -1$

$\therefore \int \frac{-1}{v} + \int \frac{1}{v+1} + \int \frac{1}{v-1}$

$= -\ln v + \ln(v+1) + \ln(v-1)$

$-\ln v + \ln(v+1) + \ln(v-1) = \ln x + \ln A$ where $C = \ln(A)$

$\ln\left[\frac{(v+1)(v-1)}{v}\right] = \ln(Ax)$