$y^{-2} = 2x + Cx^2$

(iii) $x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$

divide both sides by x^2

$\frac{dy}{dx} = x \sin 3x + \frac{4}{x^2}$

taking the integral of both sides

$y = \int x \sin 3x + \frac{4}{x^2} \, dx$

$\int x \sin 3x$

using product rule

$\int v\,du = uv - \int u\,dv$

$v = x$

$du = \sin 3x$

$dv = 1$

$u = -\frac{\cos 3x}{3}$

$= -\frac{x\cos 3x}{3} - \int -\frac{\cos 3x}{3} \times 1$

$= -\frac{x\cos 3x}{3} + \frac{\sin 3x}{9}$

$y = -\frac{x\cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1} + C$

iv) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

$\frac{dy}{dx} = \frac{2y^3}{x^3 + xy^2}$

let $y = vx$

$\frac{dy}{dx} = \frac{2(vx)^3}{x^3 + x(vx)^2}$

$\frac{dy}{dx} = \frac{2v^3(x^3)}{x^3 + vx(x^2)}$ $\frac{dy}{dx} = \frac{2v^3(x^3)}{1+v^2(x^3)}$

$\frac{dy}{dx} = x\,dv/dx + v$

$x\,dv/dx + v = \frac{2v^2}{x + vx}$

$x\,dv/dx = \frac{2v^2}{x+vx} - v$

$x\,dv/dx = \frac{2v^2 - vx}{x+vx}$

$x\,dv/dx + v = \frac{2v^3}{1+v^2}$

$x\,dv/dx = \frac{2v^3}{1+v^2} - v$

$x\,dv/dx = \frac{2v^3 - v - v^3}{1+v^2}$

$x\,dv/dx = \frac{v^3 - v}{1+v^2}$

$\frac{1+v^2}{v^3 - v} dv = \frac{1}{x} dx$

using partial fractions for the L.H.S

$-\ln v + \frac{1}{2}\ln(v-1) + \frac{1}{2}\ln(v+1) = \ln x + C$

since $y = vx$

$v = y/x$

$-\ln y/x + \frac{1}{2}\ln(y/x - 1) + \frac{1}{2}\ln(y/x + 1) = \ln x + C$

let $C = \ln A$

$\ln\left((y/x)^{-1} \times (y/x - 1)^{1/2} \times (y/x + 1)^{1/2}\right) = \ln(Ax)$

~~taking the ln of both~~

dividing through by ln

$(y/x)^{-1} \times (y/x - 1)^{1/2} \times (y/x + 1)^{1/2} = Ax$

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18/ENG05/028
Mechatronics Engineering

1) $\frac{dy}{dx} + 2y = e^{3x}$

$P = 2$

$Q = e^{3x}$

let $I.F = e^{\int p dx}$

$e^{\int 2 dx}$

so $\int 2dx = 2x$

$e^{2x} = e^{\int p dx}$

let.

$y.IF = \int Q.I.F\, dx$

$y(e^{2x}) = \int e^{3x}.e^{2x} dx$

$y(e^{2x}) = \int e^{5x} dx$

$y(e^{2x}) = \frac{e^{5x}}{5} + c$

$y = \frac{e^{5x}}{5e^{2x}} + \frac{c}{e^{2x}}$

2) $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = \frac{x^2}{x} + \frac{2x}{x} - \frac{3}{x}$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

integrating both sides

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

3) $\frac{dy}{dx} + y/x = y^3$

divide through by y^3

$y^{-3} dy/dx + \frac{y^{-2}}{x} = 1$

let $z = y^{1-n}$

$z = y^{1-3}$

$z = y^{-2}$

$\frac{dz}{dx} = -2y^{-3} dy/dx$

multiply through by -2

$-2y^{-3} dy/dx + (-2)\frac{y^{-2}}{x} = -2$

substitute z and $\frac{dz}{dx}$

$\frac{dz}{dx} - \frac{2z}{x} = -2$

let $p = -2/x$ and $Q = -2$

let $I.F = e^{\int p dx}$

$\int p dx = \int -2/x\, dx$

$= -2\ln x$

$I.F = e^{\int p dx} = e^{\ln x^{-2}} = x^{-2}$

let $z.I_f = \int Q.I.F.\,dx$

$z(x^{-2}) = \int -2 \times x^{-2} dx$

$z(x^{-2}) = \int -2x^{-2} dx$

$z(x^{-2}) = \frac{2x^{-1}}{1} + c$

$z(x^{-2}) = 2x^{-1} + c$

$z = \frac{2(x)^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

since $z = y^{-2}$