

$$b \quad \frac{dy}{dx} + 2y = e^{3x}$$

$$P = 2 \quad \int P dx = 2x$$

$$Q = e^{3x}$$

$$I.F. = e^{\int P dx} = e^{2x}$$

$$y \cdot I.F. = \int Q \cdot I.F. dx$$

$$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} dx$$

$$y \cdot e^{2x} = \frac{1}{5} e^{5x} + C$$

$$y = \frac{\frac{1}{5} e^{5x} + C}{e^{2x}}$$

$$c \quad \frac{dy}{dx} = x^2 + 2x - 3$$

$$\frac{dy}{dx} = x^2 + 2x - 3/x$$

$$\int \frac{dy}{dx} = \int (x^2 + 2x - 3/x) dx$$

$$y = x \frac{x^2}{2} + 2x = 3 \ln x + C$$

$$d \quad \frac{dy}{dx} + \frac{y}{x} = y^3$$

$$\frac{dx}{dy} y^{-3} + y^{-3}/x = 1 \dots \textcircled{1}$$

$$z = y^{1-n} \quad n = 3$$

$$z = y^{1-3} = z = y^{-2} \dots \textcircled{2}$$

$$\frac{dx}{dy} = 2y^{-3} \frac{dy}{dx} \dots \textcircled{3}$$

multiplying (eqn) and $y^{(1-n)}$

$$-2y^{-3} \frac{dy}{dx} - 2y \frac{z}{x} = -1$$

$$\text{and } dx/dy = -2y^{-3} \frac{dy}{dx}$$

sub eqn 2 and 3 into (1)

$$\frac{dx}{dy} - 2x/x = -2$$

$$\therefore P = -2/x \quad Q = -2$$

$$\int \frac{dx}{x} = -2 \ln x$$

$$\text{If } e^{-2 \ln x} = x^{-2}$$

$$z \cdot k = \int \phi \cdot k \cdot dx$$

$$\begin{aligned} z \cdot x^{-2} &= \int -2x^{-2} dx \\ &= \frac{-2x^{-1}}{-1} + C \end{aligned}$$

$$2x^{-1} = 2x^{-1} + C$$

$$z = \frac{2x^{-1}}{x^{-2}} + \frac{C}{x^{-2}}$$

$$z = 2x + Cx^2$$

$$z = x(2 + Cx)$$

$$z = y^{-2}$$

$$y^{-2} = x(2 + Cx)$$

$$\frac{y^2}{y^4} = x(2 + Cx)$$

$$y^2 = \frac{1}{x}(2 + Cx)$$

$$y = \sqrt{\frac{1}{x}(2 + Cx)}$$

e) ~~$x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$~~

$$\frac{dy}{dx} = x \sin 3x + 4$$

$$\int \frac{dy}{dx} = \int x \sin 3x + \int 4x^{-2}$$

$$= \frac{1}{3} \cos 3x + \int \frac{1}{3} \cos 3x + 4x^{-1}$$

$$= \frac{x \cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1}$$

$$y = \frac{\sin 3x}{9} - \frac{x \cos 3x}{3} - \frac{4}{x}$$

f) $(x^3 + xy^2) \frac{dy}{dx} = 2xy^2$

$$y = vx$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$f) \quad x^2 + x \frac{dy}{dx} = \frac{2y^2}{x^3 + xy^2}$$

$$= v + x \frac{dv}{dx} = \frac{2(vx)^3}{x^3 + x(vx)^2}$$

$$v + x \frac{dv}{dx} = \frac{2(vx)^3}{x^3 + x(vx)^2}$$

$$v + x \frac{dv}{dx} = \frac{2(vx)^3}{(x^3 + x(vx)^2)}$$

$$v + x \frac{dv}{dx} = \frac{2v^2 x^3}{x^3 + v^2 x^3}$$

$$v + x \frac{dv}{dx} = \frac{2x^3 (2v^3)}{x^3 (1 + v^2)}$$

$$v + x \frac{dv}{dx} = \frac{2v^3}{1 + v^2}$$

$$\frac{1 + v^2 dv}{v^3 - v} = \frac{1}{x} dx$$

$$\int \frac{1 + v^2}{v^3 - v} = \int \frac{1}{x} dx$$

$$= \ln v + \ln(v-1) + \ln(v+1) + \ln x + c$$

$$= 2 \ln\left(\frac{y}{x}\right) + \ln\left(\frac{y}{x} + 1\right) + \ln\left(\frac{y}{x} - 1\right)$$

$$= \ln x + c$$