When $V=0$

$1 = A(1)(-1) + B(0) + C(0)$

$A = -1$

therefore we have

$$\int \frac{-1}{v} + \int \frac{1}{v+1} + \int \frac{1}{v-1}$$

$-\ln(v) + \ln(v+1) + \ln(v-1)$

Solving all together

$-\ln(v) + \ln(v+1) + \ln(v-1) = \ln x + C$

Let $C = \ln A$

$$\ln\left(\frac{(v+1)(v-1)}{v}\right) = \ln x + \ln A$$

$$\ln\left(\frac{(v+1)(v-1)}{v}\right) = \ln(Ax)$$

Taking exponents of both sides

$$\frac{(v+1)(v-1)}{v} = Ax$$

Recall, $v = \frac{y}{x}$

$$\frac{v^2-1}{v} = Ax$$

$$\cancel{\frac{\frac{y^2}{x^2}-1}{y/x} = Ax} \qquad \frac{\frac{y^2}{x^2}}{y/x} - 1 = Ax$$

$$\frac{x\left(\frac{y^2}{x^2}-1\right)}{y} = Ax$$

$$\frac{x\left(\frac{y^2}{x^2}-1\right)}{y} = Ax$$

$$\left(\frac{y^2}{x} - x\right) = Axy$$

$y^2 - x^2 = Ax^2y$

$y^2 - x^2 = Ax^2C$

ENG 282

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18/ENG06/007

Mechanical Engineering

Assignment

1) $\frac{dy}{dx} + y\tanh x = 2\sinh x$

Using Integrating Factor

$I.F = e^{\int p\,dx}$

$I.F = e^{\int \tanh x\,dx}$

$I.F = e^{\int \frac{\sinh x}{\cosh x}dx} = e^{\ln(\cosh x)}$

$I.F = \cosh x$

$y \cdot IF = \int Q \cdot IF$

$y \cdot \cosh x = \int 2\sinh x \cosh x$

Integrating R.H.S

$\int 2\sinh x \cosh x$

Let $u = \cosh x$

$\frac{du}{dx} = \sinh x$, $dx = \frac{du}{\sinh x}$

$\int 2\sinh x \cdot u \cdot \frac{du}{\sinh x}$

$\int 2u\,du$

$= \frac{\cancel{2}u^2}{\cancel{2}}$

$= u^2 = \cosh^2 x$

Therefore for the full equation;

$y \cdot \cosh x = \cosh^2 x + C$

$$y = \frac{\cosh^2 x}{\cosh x} + \frac{C}{\cosh x}$$

$y = \cosh x + C\cosh^{-1} x$

2) $\frac{dy}{dx} + 2y = e^{3x}$

Using Integrating Factor

$I.F = e^{\int p\,dx} = e^{\int 2dx} = e^{2x}$

$y \cdot IF = \int Q \cdot IF$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x}$

$ye^{2x} = \int e^{3x}$

$$ye^{2x} = \frac{e^{3x}}{5} + C$$

$$y = \frac{e^{3x}}{5} + \frac{C}{e^{2x}}$$ //

3) $x\frac{dy}{dx} = x^2 + 2x - 3$

Using Direct Integration

$$\frac{dy}{dx} = \frac{x^2}{x} + \frac{2x}{x} - \frac{3}{x}$$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$\int dy = \int \left(x + 2 - \frac{3}{x}\right)dx$

$y = \int \left(x + 2 - \frac{3}{x}\right)dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + C$

4) $\frac{dy}{dx} + \frac{y}{x} = y^3$

Using Bernoulli's ~~eq~~

$z = y^{1-n}$, $z = y^{1-3}$, $z = y^{-2}$

$\frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx} = 1-3y^{-3}\frac{dy}{dx} = -2y^{-3}\frac{dy}{dx}$ 4

From the main equation

$\frac{dy}{dx} + \frac{y}{x} = y^3$

$= y^{-3}\frac{dy}{dx} + \frac{1}{xy^2} = 1$

Multiplying through by -2

$-2y^{-3}\frac{dy}{dx} - \frac{2}{xy^2} = -2$

$-2y^{-3}\frac{dy}{dx} - \frac{2}{x}y^{-2} = -2$

$\frac{dz}{dx} - \frac{2}{x}z = -2$ --> Solving using Integrating Factor

$IF = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = x^{-2}$

$x^{-2} \cdot z = \int -2 \cdot x^{-2}$

$zx^{-2} = \frac{-2x^{-2+1}}{-2+1} = 2x^{-1} + C$

Recall $z = y^{-2}$

$y^{-2}x^{-2} = 2x^{-1} + C$

$y^2 = \frac{x^{-2}}{2x^{-1}+C}$

$y^2[2x^{-1}+C] = x^{-2}$

Dividing through by x^{-2}, we have

$y^2x^2[2x^{-1}+C] = 1$

$y^2x^2 2x^{-1} + y^2x^2C = 1$

$y^2x2 + y^2x^2C = 1$

$xy^2[Cx + 2] = 1$

5) $x^2\frac{dy}{dx} = x^3\sin 3x + 4$

Direct Integration

$\frac{dy}{dx} = x\sin 3x + \frac{4}{x^2}$

$\int dy = \int \left(x\sin 3x + \frac{4}{x^2}\right)dx$

$y = \int x\sin 3x + \int \frac{4}{x^2}$

Integrating ~~$\sin 3x$~~ $x\sin 3x$ Using Integration by parts

$\int x \sin 3x$ and $\int u\,dv = uv - \int v\,du$

$u = x$ $du = 1$

$dv = \sin 3x$ $v = -\frac{\cos 3x}{3}$

$= -\frac{x\cos 3x}{3} - \int -\frac{\cos 3x}{3} \cdot 1$

$= \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9}$

Placing everything together

$y = \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} + \frac{4x^{-2+1}}{-2+1}$

$y = \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} - \frac{4}{x} + C$ //

6) $(x^3 + xy^2)\frac{dy}{dx} = 2y^3$

Using Homogenous Method

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\frac{dy}{dx} = \frac{2y^3}{x^3+xy^2}$

$\frac{dy}{dx} = \frac{2(vx)^3}{x^3+x(vx)^2}$

$\frac{dy}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$

$\frac{dy}{dx} = \frac{2v^3}{1+v^2}$

$v + x\frac{dv}{dx} = \frac{2v^3}{1+v^2}$

$x\frac{dv}{dx} = \frac{2v^3}{1+v^2} - v$

$x\frac{dv}{dx} = \frac{2v^3 - v(1+v^2)}{1+v^2}$

$x\frac{dy}{dx} = \frac{2v^3 - v - v^3}{1+v^2}$

$x\frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$

Separating Variables we have

$\int \frac{1+v^2}{v^3-v}dv = \int \frac{1}{x}dx$

Integrating LHS Using Partial Fractions

$\frac{1+v^2}{v^3-v} = \frac{1+v^2}{v(v^2-1)} = \frac{1+v^2}{(v)(v+1)(v-1)}$

$\frac{1+v^2}{(v)(v+1)(v-1)} = \frac{A}{v} + \frac{B}{v+1} + \frac{C}{v-1}$

$1+v^2 = A(v+1)(v-1) + B(v)(v-1) + C(v)(v+1)$

When $v = 1$

$2 = A(0) + B(0) + C(1)(2)$

$2 = 2C$

$C = 1$

When $v = -1$

$2 = A(0) + B(-1)(-2) + C(0)$

$2 = 2B$

$B = 1$