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18/ENG02/002
COMP. ENGINEERING

1 $\frac{dy}{dx} + y\tanh x = 2\sinh x$

$\frac{dy}{dx} + Py = Q$

$y.\,If = \int I.F \times Q\,dx$

$I.f = e^{\int P dx}$

$P = \tanh x$

$Q = 2\sinh x$

$I.f = e^{\int \tanh x\,dx}$

but $\tanh x = \frac{\sinh x}{\cosh x}$

$I.F = e^{\int \frac{\sinh x}{\cosh x} dx}$

$I.F = e^{\ln \cosh x}\,dx$

$I.F = \cosh x$

$y.\cosh x = \int 2\sinh x \cosh x\,dx$

$y\cosh x = 2\int \sinh x \cosh x\,dx$

$y\cosh x = 2 \times \left(\frac{\cosh x^2}{2}\right) + c$

$y = \cosh x + \frac{c}{\cosh x}$

b $\frac{dy}{dx} + 2y = e^{3x}$

$\frac{dy}{dx} + Py = Q$

$P = 2 \quad Q = e^{3x}$

$I.f = e^{\int P dx}$

$= e^{\int 2 dx}$

$If = e^{2x}$

$y.\,If = \int Q.\,I.F\,dx$

$y \times e^{2x} = \int e^{3x} \times e^{2x} dx$

$y \times e^{2x} = \int e^{5x}\,dx$

$ye^{2x} = \frac{1}{5} e^{5x} + c$

$y = \frac{1}{5} e^{3x} + ce^{-2x}$

C $x\frac{dy}{dx} = x^2 + 2x - 3$

$\frac{dy}{dx} = x + 2 - \frac{3}{x}$

$dy = \left(x + 2 - \frac{3}{x}\right)dx$

$\int dy = \int x + 2 - \frac{3}{x}\,dx$

$y = \frac{x^2}{2} + 2x - 3\ln x + c$

d $\frac{dy}{dx} + \frac{y}{x} = y^3$

$P = \frac{1}{x} \qquad Q = 1$

$\frac{dz}{dx} + P_1 z = Q_1$

$z = y^{1-n} = z = y^{1-2} = y^{-2}$

$P_1 = \frac{1}{x}(1-3) = -\frac{2}{x}$

$z \cdot I.f = \int Q_1 \times IF \, dx$

$I.F = e^{\int P_1 dx}$

$e^{\int -2/x \, dx}$

$e^{-2\ln x}$

$I.F = x^{-2}$

$z \times x^{-2} = \int -2 \times x^{-2} dx$

$z \times x^{-2} = \frac{2}{x} + c$

$y^2 = \frac{1}{2x + cx^2}$

(e) $x^2 \frac{dy}{dx} = x^3 \sin 3x + 4$

$\frac{dy}{dx} = x \sin 3x + 4x^{-2}$

$dy = x \sin 3x + 4x^{-2} dx$

$\int dy = \int x \sin 3x + 4x^{-2} dx$

$y = \int x \sin 3x \, dx + \int 4x^{-2} dx$

$\int 4x^{-2} dx = \frac{4x^{-1}}{-1} = -\frac{4}{x}$

$\int x \sin 3x \, dx = \int u \, dv$

$\int u \, dv = uv - \int v \, du$

$u = x, \; dv = \sin 3x, \; du = 1$

$v = \int \sin 3x \, dx = -\frac{1}{3} \cos 3x$

$\int x \sin 3x \, dx = x \times -\frac{1}{3} \cos 3x$

$- \int -\frac{1}{3} \cos 3x \, dx$

$= -\frac{x \cos 3x}{3} + \left(\frac{1}{3} \cdot \frac{+1}{3} \times \sin 3x\right)$

$= -\frac{x \cos 3x}{3} + \frac{\sin 3x}{9}$

$y = \frac{\sin 3x}{9} - \frac{x \cos 3x}{3} - \frac{4}{x} + c$

(f) $(x^2 + xy^2) \frac{dy}{dx} = 2y^2$

$\frac{dy}{dx} = \frac{2y^3}{x^3 + xy^2}$

let $y = vx$

$\frac{dy}{dx} = v + x \frac{dv}{dx}$

$v + \frac{x dv}{dx} = \frac{2(vx)^3}{x^3 + x(vx)^2}$

$v + \frac{x dv}{dx} = \frac{2v^3 x^3}{x^3 + v^2 x^3}$

$v + \frac{x dv}{dx} = \frac{2v^3 x^3}{x^3(1+v^2)}$

$x \frac{dv}{dx} = \frac{v^3 - v}{1 + v^2}$

$\frac{1+v^2}{v^3 - v} \, dv = \frac{dx}{x}$

$\int \frac{1+v^2}{v^3 - v} \, dv = \int \frac{dx}{x}$

but $\frac{1+v^2}{v^3 - v} = \frac{1+v^2}{v(v^2-1)} = \frac{1+v^2}{v(v-1)(v+1)}$

$\frac{1+v^2}{v^3 - v} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$

$\frac{1+v^2}{v^3 - v} = \frac{A(v^2-1) + B(v^2+v) + C(v^2-v)}{v(v-1)(v+1)}$

At $v = 1$, $1 + (1)^2 = B(1^2+1)$

$B \times 2 = 2 \quad \therefore B = 1$

At $v = -1$

$1 + (-1)^2 = A((-1)^2 - 1) + B((-1)^2 + (-1)) + C((-1)^2 + 1)$

$1 + 1 = C$

$2 = C \times 2$

$C = 1$

$1 + v^2 = A(v^2 - 1) + (v^2 + v) + 1(v^2 - v)$

$1 + 1 = A(v^2 - 1) + v^2$

let $v = 2$

$1 = A(3) + 4$

$A = -1$

$\frac{1+v^2}{v^3 - v} = \frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1}$

$= \int \frac{-1}{v} + \frac{1}{v-1} + \frac{1}{v+1} \, dv$

$= \ln \frac{v^2 - 1}{v}$

$\int \frac{dx}{x} = \ln x + c$

$\ln\left(\frac{v^2 - 1}{v}\right) = \ln x + c$

$v = \frac{y}{x}$

$\ln\left(\frac{v^2 - 1}{v}\right) = \ln\left(\frac{\frac{y^2}{x^2} - 1}{y/x}\right)$

$= \ln\left(\frac{y^2 - x^2}{xy}\right)$

$\frac{y^2 - x^2}{xy} = e^{\ln x + c}$

where $A = e^c$

$\frac{y^2 - x^3}{xy} = Ax$

$y^2 = Ax^2 y + x^2$