$y^{-2} = x(2+cx)$

$1/y^2 = x(2+cx)$

$y^2 = 1/x(2+cx)$

$\therefore y = \sqrt{1/x(2+cx)}$

(2) $x^2 \, dy/dx = x^3 \sin 3x + 4$

$dy/dx = x \sin 3x + 4$

$\int dy/dx = \int x \, 4 \sin 3x + \int 4x^{-2}$

$= 1/3 \cos 3x + \int 1/3 \cos 3x + 4x^{-1}$

$= \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} - 4x^{-1}$

$y = \frac{\sin 3x}{4} - \frac{x\cos 3x}{3} - \frac{4}{x}$

(f) $(x^3 + xy^2)\, dy/dx = 2y^3$

$y = vx$

$dy/dx = v + x \, dv/dx$

$\frac{dy}{dx} = \frac{2y^3}{(x^3+xy^2)}$

$= v + x\frac{dv}{dx} = \frac{2(vx)^3}{(x^3 + x(vx)^2)}$

$v + x\frac{dv}{dx} = \frac{2v^3x^3}{x^3 + v^2x^3}$

$\therefore v + x\frac{dv}{dx} = \frac{x^3(2v^3)}{x^3(1+v^2)}$

$v + x\frac{dv}{dx} = \frac{2v^3}{1+v^2}$

$x\frac{dv}{dx} = \frac{2v^3 - (v+v^3)}{1+v^2}$

$= x\frac{dv}{dx} = \frac{v^3 - v}{1+v^2}$

$\frac{1+v^2 \, dv}{v^3 - v} = \frac{1}{x} dx$

$\int \frac{1+v^2}{v^3-v} = \int 1/x \, dx$

$= -\ln(v) + \ln(v-1) + \ln(v+1) = \ln x + c$

$= -\ln(y/x) + \ln(y/x - 1) + \ln(y/x + 1)$

$= \ln x + c$

1. $\frac{dy}{dx} = 2\sinh x - y\tanh x$

$\frac{dy}{dx} + y\tanh x = 2\sinh x$

$P = \tan x$

$\int P dx = \int \tanh x = \int \frac{\sinh x}{\cosh x} dx$

$\cosh x = u \quad \frac{du}{dx} = \sinh x$

$dx = du / \sinh x$

$\int \frac{\sinh x}{u} \cdot du/\sinh x = \int 1/u \, du = \ln u = \ln\cosh x$

$IF = e^{\int P dx} = e^{\ln\cosh x} \quad IF = \cosh x$

$y \cdot IF = \int Q \, IF \, dx$

$y = \cosh x = \int 2\sinh x \cdot \cosh x \, dx$

$2\sin x\cos x = \sin(2x)$

$\therefore 2\sinh x\cos x = \sinh(2x)$

$y \cdot \cosh x = \int \sinh 2x \, dx$

$y \cdot \cosh x = 1/2 \cdot \cosh 2x + c$

$\cosh x y = \cosh 2x + c$

$y = \frac{\frac{\cosh 2x + c}{2}}{\cosh x}$

$y = \frac{\cosh 2x + 2c}{\cosh x}$

let $2c = A$

$y = \frac{\cosh 2x + A}{\cosh x}$

b) $dy/dx + 2y = e^{3x}$

$P = 2 \quad \int P dx = 2x$

$Q = e^{3x}$

$IF = e^{\int P dx} = e^{2x}$

$y \cdot IF = \int Q \cdot IF \, dx$

$y \cdot e^{2x} = \int e^{3x} \cdot e^{2x} \, dx$

$y \cdot e^{2x} = \int e^{5x} \, dx$

$y \cdot e^{2x} = 1/5 e^{5x} + c$

$y = \frac{1/5 e^{5x} + c}{e^{3x}}$

c) $2 \, dy/dx = x^2 + 2x - 3$

$dy/dx = x + 2 - 3/x$

$\therefore \int dy/dx = \int x + 2 - 3/x \, dx$

$y = x^2/2 + 2x - 3\ln x + c$

d) $dy/dx + y/x = y^3$

$dy/dx \, y^{-3} + y^{-3}/x = 1$ - - - (i)

$z = y^{1-n}, \; n = 3$

$z = y^{1-3}, \; z = y^{-2}$ - - - - (ii)

$dz/dy = 2y^{-3} \, dy/dx$ - - - - (iii)

Then multiplying equation 1 by $y(1-n)$

$-2y^{-3}\frac{dy}{dx} - 2y^{-2}/x = -2$

and $dz/dy = -2y^{-3} \, dy/dx$

Sub equation (ii) and (iii) into (iv)

$dz/dy - 2z/x = -2$

$\therefore P = -2/x \quad Q = -2$

$\int P dx = -2\ln x$

$IF = e^{-2\ln x} = x^{-2}$

$z \cdot IF = \int Q \cdot IF \cdot dx$

$z \cdot x^{-2} = \int -2x^{-2} dx$

$= \frac{-2x^{-1}}{1} + c$

$2x^{-1} = 2x^{-1} + c$

$z = \frac{2x^{-1}}{x^{-2}} + \frac{c}{x^{-2}}$

$z = 2x + cx^2$

$z = x(2 + cx)$

$\Rightarrow = y^{-2}$