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MAT 104 Assignment: For the curves in problem 1 to 5, at the points given, find (a) the equation of the tangent, and (b) the equation of the normal.

1. $y = 2x^2$ at point $(1, 2)$
2. $y = 3x^2 - 2x$ at the point $(2, 8)$
3. $y = x^3/2$ at the point $(-1, -1/2)$
4. $y = 1 + x - x^2$ at the point $(-2, -5)$
5. $y = 1/x$ at the point $(3, 1/3)$

Solution

1) $y = 2x^2$ at the point $(1, 2)$
 $\frac{dy}{dx} = 4x$ $\frac{dy}{dx}|_{x=1} = 4(1)$

$m = 4, \quad y_1 = 2, \quad x_1 = 1$

Equation of tangent

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 4(x - 1)$$

$$y - 2 = 4x - 4$$

$$y - 4x + 2 = 0$$

Equation of the normal

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - 2 = -\frac{1}{4}(x - 1)$$

$$4y - 8 = -x + 1$$

$$4y + x - 9 = 0$$

2) $y = 3x^2 - 2x$ at the point $(2, 8)$
 $\frac{dy}{dx} = 6x - 2$ $\frac{dy}{dx}|_{x=2} = 6(2) - 2$

$m = 10, \quad x_1 = 2, \quad y_1 = 8$

$$y - y_1 = m(x - x_1)$$

$$y - 8 = 10(x - 2)$$

$$y - 8 = 10x - 20$$

$$y - 10x + 12 = 0$$

Equation of tangent

Equation of the normal.

$$y - y_1 = -\frac{1}{m_1} (x - x_1)$$

$$y - 8 = -\frac{1}{10} (x - 2)$$

$$10y - 80 = -x + 2$$

$$10y + x - 82 = 0$$

3) $y = x^3/2$ $(-1, -1/2)$

$$\frac{dy}{dx} = \frac{x^3}{2} = \frac{3x^2}{2} = \frac{3(-1)^2}{2}$$

$$m = \frac{3}{2}$$

$$y - y_1 = m (x - x_1)$$

$$y + \frac{1}{2} = \frac{3}{2} (x + 1)$$

$$2y + 1 = 3x + 3 \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{ Divide through by 2}$$

$$y + \frac{1}{2} = \frac{3x + 3}{2}$$

$$y - \frac{3x}{2} - \frac{1}{2} = 0 \quad \text{Equation of tangent}$$

Equation of normal.

$$y - y_1 = -\frac{1}{m_1} (x - x_1)$$

$$y + \frac{1}{2} = -\frac{1}{3/2} (x + 1)$$

$$y + \frac{1}{2} = -\frac{2}{3} (x + 1)$$

$$3y + \frac{3}{2} = -2x - 2$$

$$3y + 2x + 7 = 0$$

4) $y = 1 + x - x^2$ $(-2, -5)$

$$\frac{dy}{dx} = 1 - 2x$$

$$\frac{dy}{dx} \bigg|_{x=-2} = -2$$

$$1 - 2(-2)$$

$$m = 5$$

$$y - y_1 = m (x - x_1)$$

$$y + 5 = 5 (x + 2)$$

$$= 5y + 5 = 5x + 10$$

$$y - 5x - 5 = 0 \quad \text{equation of tangent}$$

Equation of normal

$$y - y_1 = -\frac{1}{m_1} (x - x_1)$$

$$y + 5 = -\frac{1}{5} (x + 2)$$

$$5y + 10 = -x - 2$$

$$5y + x + 12 = 0$$

$$5) \quad y = \frac{1}{x} = x^{-1} \quad \frac{dy}{dx} = -x^{-2}$$

$$\frac{dy}{dx} \Big|_{x=3} = -(3)^{-2} = -\frac{1}{9}$$

$$m = -\frac{1}{9}, \quad y = \frac{1}{3}, \quad x = 3$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{3} = -\frac{1}{9}(x - 3)$$

$$9y - 9 = -3x + 9$$

$$9y + 3x - 18 = 0 \quad \text{Divide through by 3}$$

$$3y + x - 6 = 0 \quad \text{Equation of tangent}$$

Equation of normal

$$y - y_1 = -\frac{1}{m_1}(x - x_1)$$

$$y - \frac{1}{3} = -\frac{1}{-\frac{1}{9}}(x - 3)$$

$$y - \frac{1}{3} = 9(x - 3)$$

Multiply through by 3

$$3y - 1 = 27x - 81$$

$$3y - 27x + 80 = 0 //$$