MBBS

Name: ESE-EJAKPOVI UYOYOGHENE

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COURSE CODE: MAT 104 ASSIGNMENT

100 Level.

ASSIGNMENT ~~TXTL~~ TITLE: Equation of tangent and normal.

1. $y = 2x^2$ at the point $(1, 2)$.

Solution:

$\frac{dy}{dx} = 4x$ at $x = 1$

$m = 4(1)$

$\therefore m = 4$ (gradient).

(a) $(y - y_1) = m(x - x_1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y - 4x - 2 + 4 = 0$

$\therefore$ The equation of the tangent is

$y - 4x + 2 = 0$.

(b) Equation of the normal:

$y - y_1 = \frac{-1}{m}(x - x_1)$

$\frac{y-2}{1} = \frac{-1}{4}(x - 1)$

$4(y-2) = -1(x-1)$

$4y - 8 = -x + 1$

$4y + x - 8 - 1 = 0$

$\therefore$ the equation of the normal is $4y + x - 9 = 0$.

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(2) $y = 3x^2 - 2x$ at point $(2,8)$.

Solution:

$$\frac{dy}{dx} = 6x - 2 \quad \text{at } x = 2$$

$$m = 6(2) - 2$$
$$= 12 - 2$$
$\therefore m$ (gradient) $= 10$.

(a) The equation of the tangent:

$$(y - y_1) = m(x - x_1)$$
$$y - 8 = 10(x - 2)$$
$$y - 8 = 10x - 20$$
$$\cancel{y} \; y - 10x - 8 + 20 = 0$$

$\therefore$ the equation of the tangent is $y - 10x + 12 = 0$.

(b) Equation of the normal:

$$(y - y_1) = -\frac{1}{m}(x - x_1)$$

$$(y - 8) = \frac{-1}{10}(x - 2)$$

$$10(y - 8) = -1(x - 2)$$
$$10y - 80 = -x + 2$$
$$10y + x - 80 - 2 = 0$$

$\therefore$ The equation of the normal is

$10y + x - 82 = 0$.

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(3.) $y = \frac{x^3}{2}$ at point $(-1, -1/2)$

Solution:

$$y = \frac{x^3}{2}$$

$u = x^3 \quad du/dx = 3x^2$

$v = 2 \quad dv/dx = 0$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{6x^2 - 0}{4}$$

$$= \frac{\cancel{6}^{3}x^2}{\cancel{4}_{2}}$$

$$\therefore \frac{dy}{dx} = \frac{3x^2}{2} \quad \text{at } x = -1$$

$$m = \frac{3(-1)^2}{2}$$

$$\therefore m = \frac{3}{2}$$

(a) Equation of the tangent.

$$(y - y_1) = m(x - x_1)$$

$$y + \frac{1}{2} = \frac{3}{2}(x+1)$$

$$\frac{2y+1}{2} = \frac{3}{2}(x+1)$$

$$2(2y+1) = 6(x+1)$$

$$4y + 2 = 6x + 6$$

$4y - 6x + 2 - 6 = 0$

$4y - 6x - 4 = 0$

$\therefore$ The equation of the tangent is

$4y - 6x - 4 = 0$.

(b) Equation of the normal.

$(y - y_1) = \frac{-1}{m}(x - x_1)$

$(y + \frac{1}{2}) = \frac{-2}{3}(x + 1)$

$\frac{2y + 1}{2} = \frac{-2x - 2}{3}$

$3(2y + 1) = 2(-2x - 2)$.

$6y + 3 = -4x - 4$

$6y + 4x + 3 + 4 = 0$

$\therefore$ The equation of the normal is

$6y + 4x + 7 = 0$.

(4.) $y = 1 + x - x^2$ at the point $(-2, -5)$

Solution:

$\frac{dy}{dx} = 1 - 2x$, at $x = -2$

$m = 1 - 2(-2)$

$= 1 + 4$

$\therefore m = 5$.

(a) Equation of the tangent

$(y-y_1) = m(x-x_1)$

$(y+5) = 5(x+2)$

$y+5 = 5x+10$

$y-5x+5-10=0$

$\therefore$ the equation of the tangent is $y-5x-5=0$.

(b) Equation of the normal

$(y-y_1) = \frac{-1}{m}(x-x_1)$

$\frac{(y+5)}{1} \times \frac{-1}{5}(x+2)$

$5(y+5) = -x-2$

$5y+25+x+2=0$

$\therefore$ The equation of the normal is

$5y+x+27=0$.

(5.) $y = \frac{1}{x}$ at point $(3, 1/3)$.

Solution:

$y = \frac{1}{x}$

$u = 1$, $du/dx = 0$

$v = x$, $dv/dx = 1$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{0-1}{x^2}$$

$$\therefore \frac{dy}{dx} = \frac{-1}{x^2} \text{ at point } x = 3$$

$$\therefore m = \frac{-1}{3^2}$$

$$= \frac{-1}{9}$$

ⓐ Equation of the tangent

$$(y - y_1) = m(x - x_1)$$

$$(y - \tfrac{1}{3}) = \frac{-1}{9}(x-3)$$

$$\frac{3y-1}{3} \times \frac{-x+3}{9}$$

$$9(3y-1) = 3(-x+3)$$

$$27y - 9 = -3x + 9$$

$$27y + 3x - 9 - 9 = 0$$

$\therefore$ the equation of the tangent is

$$27y + 3x - 18 = 0 \text{ (multiples of 3)}$$

OR $$9y + x - 6 = 0$$

ⓑ Equation of the normal.

$$(y - y_1) = \frac{-1}{m}(x - x_1)$$

$$(y - \tfrac{1}{3}) = 9(x-3)$$

$$\frac{3y-1}{3} \times \frac{9(x-3)}{1}$$