PINNICK TISE ORITSETSERUNDEDE
CHEMICAL ENGINEERING
19/ENG01/013
SERIAL NUMBER : 48
MAT 204 Assignment

Differentiate the following

Question 1

$$y = \frac{[(x+1)^2 (x-2)^{1/2}]}{[(2x-1)(x+3)^{3/2}]}$$

$$\ln y = \left[2\ln(x+1) + \tfrac{1}{2}\ln(x-2)\right] - \left[\ln(2x-1) + \tfrac{3}{2}\ln(x+3)\right]$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \left(2\cdot\frac{1}{x+1}\cdot 1 + \frac{1}{2}\cdot\frac{1}{x-2}\cdot 1\right) - \left(\frac{1}{2x-1}\cdot 2 + \frac{3}{2}\cdot\frac{1}{x+3}\cdot 1\right)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \left(\frac{2}{x+1} + \frac{1}{2(x-2)}\right) - \left(\frac{2}{2x-1} + \frac{3}{2(x+3)}\right)$$

$$\frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{3}{2(x+3)}\right]$$

$$\therefore \frac{dy}{dx} = \frac{[(x+1)^2(x-2)^{1/2}]}{[(2x-1)(x+3)^{3/2}]}\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{3}{2x+3}\right]$$

Question 2

$$y = \frac{3e^x \sin 2x}{x^{5/2}}$$

$$\ln y = \left(\ln(3e^x) + \ln(\sin 2x)\right) - \left(\tfrac{5}{2}\ln x\right)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{3e^x}\cdot 3e^x + \frac{1}{\sin 2x}\cdot 2\cos 2x - \frac{5}{2}\cdot\frac{1}{x}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = 1 + \frac{2\cos 2x}{\sin 2x} - \frac{5}{2x}$$

$$\frac{dy}{dx} = y\left[1 + \frac{2\cos 2x}{\sin 2x} - \frac{5}{2x}\right]$$

$$\therefore \frac{dy}{dx} = \frac{[3e^x \sin 2x]}{x^{5/2}}\left[1 + 2\cot 2x - \frac{5}{2x}\right]$$

Integrate the following with respect to the Variable

Question 1

$$4\sec^2(3m+1)$$

$$\int 4\sec^2(3m+1)\,dm$$

$$4\int \sec^2(3m+1)\,dm$$

$$u = 3m+1$$

$$\frac{du}{dm} = 3$$

$$dm = \frac{du}{3}$$

$$4\int \sec^2 u \cdot \frac{du}{3}$$

$$\frac{4}{3}\int \sec^2 u\,du$$

$$\frac{4}{3}[\tan u] + C$$

$$\therefore \int 4\sec^2(3m+1)\,dm = \frac{4}{3}\tan(3m+1) + C$$

Question 2

$$2t(3t^2-1)^{1/2}$$

$$\int 2t(3t^2-1)^{1/2}\,dt$$

$$2\int t(3t^2-1)^{1/2}\,dt$$

$$u = (3t^2-1)^{1/2}$$

$$u^2 = 3t^2-1$$

$$u^2+1 = 3t^2$$

$$t^2 = \frac{u^2+1}{3}$$

$$t = \left(\frac{u^2+1}{3}\right)^{1/2}$$

$$\frac{dt}{du} = \frac{1}{\cancel{2}}\left(\frac{u^2+1}{3}\right)^{-1/2} \cdot \frac{\cancel{2}u}{3}$$

$$\frac{dt}{du} = \frac{u}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$dt = \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$2\int\left(\frac{u^2+1}{3}\right)^{1/2} \cdot u \cdot \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$$

$$\frac{2}{3}\int\left(\frac{u^2+1}{3}\right)^{1/2-1/2} \cdot u^2\,du$$

$$\frac{2}{3}\int u^2\,du$$

$$2/3\left[\frac{u^3}{3}\right] + C$$

$$\therefore \int 2t(3t^2-1)^{1/2}\,dt = \frac{2(3t^2-1)^{3/2}}{3} + C$$

<u>Question 3</u>

$$\frac{2x}{(4x^2-1)^{1/2}}$$

$$2\int\frac{x}{\sqrt{4x^2-1}}\,dx$$

$$u = \sqrt{4x^2-1}$$

$$u^2 + 1 = 4x^2$$

$$x = \left(\frac{u^2+1}{4}\right)^{1/2}$$

$$\frac{dx}{du} = \frac{1}{2}\left(\frac{u^2+1}{4}\right)^{-1/2} \cdot \frac{u}{2}$$

$$dx = \frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$2\int\left(\frac{u^2+1}{4}\right)^{1/2} \cdot \frac{1}{u} \cdot \frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$$

$$= \frac{2}{4}\int\left(\frac{u^2+1}{4}\right)^{1/2-1/2} \cdot \frac{1}{u}\,u\,du$$

$\frac{1}{2}\int du$

$= \frac{1}{2}[u] + C$

$\therefore \int \frac{2x}{(4x^2-1)^{1/2}}\,dx = \frac{\sqrt{4x^2-1}}{2} + C$

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