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Assignment

Find the equation of the normal and tangent of the following:

i) $y = 2x^2$ at the point (1, 2)

Solution

$y = 2x^2$

$\frac{dy}{dx} = 4x$

$m = \left.\frac{dy}{dx}\right|_{x=1} = 4(1) = 4$

∴ The equation of the tangent gives

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y - 4x - 2 + 4 = 0$

$y - 4x + 2 = 0$

The equation of the normal gives,

$y - y_1 = -1/m(x - x_1)$

$y - 2 = -\frac{1}{4}(x - 1)$

$4(y - 2) = -1(x - 1)$

$4y - 8 = -x + 1$

$4y + x - 8 - 1 = 0$

$4y + x - 9 = 0$

2. $y = 3x^2 - 2x$ at the point $(2, 8)$

Solution

$y = 3x^2 - 2x$

$\frac{dy}{dx} = 6x - 2$

$y = 3x^2 - 2x$

$\frac{dy}{dx} = 6x - 2$

$m = \left.\frac{dy}{dx}\right|_{x=2} = 6(2) - 2 = 12 - 2 = 10$

The equation of the tangent gives,

$y - y_1 = m(x - x_1)$

$y - 8 = 10(x - 2)$

$y - 8 = 10x - 20$

$y - 10x - 8 + 20 = 0$

$y - 10x + 12 = 0$

The equation of the normal gives

$y - y_1 = -1/m(x - x_1)$

$y - 8 = -\frac{1}{10}(x - 2)$

$10(y - 8) = -1(x - 2)$

$10y - 80 = -x + 2$

$10y + x - 80 - 2 = 0$

$10y + x - 82 = 0$

3. $y = \frac{x^3}{2}$ at the point $(-1, -\frac{1}{2})$

Solution

$$y = \frac{x^3}{2}$$

$$\frac{dy}{dx} = \frac{3x^2}{2}$$

$$m = \left.\frac{dy}{dx}\right|_{x=-1} = \frac{3(-1)^2}{2} = \frac{3}{2}$$

∴ The equation of the tangent gives,

$$y - y_1 = m(x - x_1)$$

$$y - (-\tfrac{1}{2}) = \frac{3}{2}(x - (-1))$$

$$y + \frac{1}{2} = \frac{3}{2}(x + 1)$$

$$2(y + \tfrac{1}{2}) = 3(x + 1)$$

$$2y - 3x = 0$$

$$\frac{y}{1} + \frac{1}{2} = \frac{3}{2}(x+1)$$

$$\frac{2y+1}{2} = \frac{3}{2}(x+1)$$

$$2(2y+1) = 2\left[3(x+1)\right]$$

$$4y + 2 = 2[3x+3]$$

$$4y + 2 = 6x + 6$$

$$4y - 6x + 2 - 6 = 0$$

$$4y - 6x - 4 = 0$$

$$2y - 3x - 2 = 0$$

The equation of the normal gives,

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - (-\tfrac{1}{2}) = -\tfrac{2}{3}(x - (-1))$$

$$\frac{y}{1} + \frac{1}{2} = -\frac{2}{3}(x+1)$$

$$\frac{2y+1}{2} = \frac{-2}{3}(x+1)$$

$$3(2y+1) = 2[-2(x+1)]$$

$$6y+3 = 2[-2x-2]$$

$$6y+3 = -4x-4$$

$$6y+4x+3+4 = 0$$

$$6y+4x+7 = 0$$

4. $y = 1+x-x^2$ at the point $(2,-5)$

Solution

$$y = 1+x-x^2$$

$$\frac{dy}{dx} = 1-2x$$

$$m = \left.\frac{dy}{dx}\right|_{x=-2} = 1-2(2) = 1+4 = 5$$

The equation of the tangent gives,

$$y-y_1 = m(x-x_1)$$

$$y-(-5) = 5(x-(-2))$$

$\left.\frac{dx}{dx}\right|_{x=-2}$

The equation of the tangent gives,

$y - y_1 = m(x - x_1)$

$y - (-5) = 5(x - (-2))$

$y + 5 = 5(x + 2)$

$y + 5 = 5x + 10$

$y - 5x + 5 - 10 = 0$

$y - 5x - 5 = 0$

The equation of the normal gives,

$y - y_1 = -1/m(x - x_1)$

$y - (-5) = -1/5(x - (-2))$

$y + 5 = \frac{-1}{5}(x + 2)$

$5(y + 5) = -1(x + 2)$

$5y + 25 = -x - 2$

$5y + x + 25 + 2 = 0$

$5y + x + 27 = 0$

5. $y = \frac{1}{x}$ at the point (3, 1/3)

Solution

$y = \frac{1}{x} = x^{-1}$

$\frac{dy}{dx} = -x^{-2} = \frac{-1}{x^2}$

$m = \left.\frac{dy}{dx}\right|_{x=3} = -3^{-2} = \frac{-1}{3^2} = -\frac{1}{9}$

∴ The equation of the tangent gives,

$y - y_1 = m(x - x_1)$

$y - 1/3 = -\frac{1}{9}(x - 3)$

$\frac{y}{1} - \frac{1}{3} = -\frac{1}{9}(x - 3)$

$\frac{3y - 1}{3} = -\frac{1}{9}(x - 3)$

$9(3y - 1) = 3[-1(x - 3)]$

$$\frac{y}{1} - \frac{1}{3} = -\frac{1}{9}(x-3)$$

$$\frac{3y-1}{3} = -\frac{1}{9}(x-3)$$

$$9(3y-1) = 3[-1(x-3)]$$

$$9(3y-1) = 3(-x+3)$$

$$27y - 9 = -3x + 9$$

$$27y + 3x - 9 - 9 = 0$$

$$27y + 3x - 18 = 0$$

$$9y + x - 6 = 0$$

∴ The equation of the normal gives,

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$\frac{y}{1} - \frac{1}{3} = 9(x-3)$$

$$\frac{3y-1}{3} = 9x - 27$$

$$3y - 1 = 3(9x - 27)$$

$$3y - 1 = 27x - 81$$

$$3y - 27x - 1 + 81 = 0$$

$$3y - 27x + 80 = 0$$