

Calculus with MATHS 104

MASH 104

MBBS

19/MYSO1/180

Find the equation of (a) Tangent (b) Normal of the following

1. $y = 2x^2$
(1, 2)

Solution

$$\frac{dy}{dx} = 4x$$

$$m = \left. \frac{dy}{dx} \right|_{x=1} = 4(1)$$

$$m_1 = 4$$

Then the equation of the tangent would be using

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 4(x - 1)$$

$$y - 2 = 4x - 4$$

$$y - 2 + 4 - 4x = 0$$

$$y + 2 - 4x = 0 \quad \text{or} \quad y - 4x = -2$$

The equation of the normal

$$m_1 m_2 = -1$$

such that

$$m_2 = \frac{-1}{m_1} = \frac{-1}{4}$$

$$y - 4 = 2 \cdot m(x - 2)$$

$$y - 2 = m(x - 1)$$

$$y - 2 = \frac{2x}{4} + \frac{1}{4}$$

Multiplying through by 4

$$4y - 8 = 2x + 1$$

$$4y + 2x = 11$$

$$4y + 2x = 9$$

$$y = 2x - 2$$

(2, 2)

Solution

$$\frac{dy}{dx} = 6x - 2$$

$$m = \frac{dy}{dx} = 6(2) - 2 = 12 - 2 = 10$$

Equation of Tangent line

$$y - 4 = m(x - 2)$$

$$y - 4 = 10(x - 2)$$

$$y - 4 = 10x - 20$$

$$y - 8 + 20 = 10x - 20 + 20 = 0$$

$$y + 12 = 10x = 0$$

Equation of Normal

$$m_2 = \frac{1}{m} = -\frac{1}{10}$$

$$y - 4 = -\frac{1}{10}(x - 2)$$

$$y - 4 = -\frac{x}{10} + \frac{1}{5}$$

multiply through by 10

$$10y - 40 = -x + 2$$

$$10y + x = 42$$

(6) $y = x - \frac{2}{3}$

(-1, 1/3)

Solution

$$\frac{dy}{dx} = 3x^2 + 2$$

$$\frac{dy}{dx} = 6x^2 + 0$$

$$\frac{dy}{dx} = 6x^2$$

$$m = \frac{dy}{dx} = 6(-1)^2 = 6$$

Equation of Tangent

$$y - 4 = m(x - 2)$$

$$y - 4 = 6(x - 2)$$

$$y + 8 = 6x - 12$$

$$2y + 1 = 6x + 2$$

$$2y + 1 - 3x = 0$$

$$2y - 3 - 3x = 0$$

Equation of the Normal

$$m_2 = -\frac{1}{m} = -\frac{1}{6}$$

$$m_2 = -\frac{1}{6}$$

$$y + 5 = -\frac{1}{5}(x + 1)$$

$$y + 5 = -\frac{x+1}{5}$$

$$5y + 25 = -x - 1$$

$$5y + x + 26 = 0$$

$$5y + x + 27 = 0$$

$$y = 1 + x - x^2$$

$$(-2, -5)$$

solution

$$\frac{dy}{dx} = 1 - 2x$$

$$\left. \frac{dy}{dx} \right|_{x=-2} = 1 - (-2) = 1 + 2 = 3$$

$$m_1 = 3$$

The equation of the tangent

$$y - y_1 = m(x - x_1)$$

$$y - (-5) = 3(x - (-2))$$

$$y + 5 = 3(x + 2)$$

$$y + 5 = 3x + 6$$

$$y + 5 - 6 - 3x = 0$$

$$y - 3x - 1 = 0$$

Equation of the Normal

$$m_2 = -\frac{1}{3}$$

$$y + 5 = -\frac{1}{3}(x + 2)$$

$$y + 5 = -\frac{x+2}{3}$$

$$3y + 15 = -x - 2$$

$$3y + x + 17 = 0$$

$$3y + x + 17 = 0$$

$$\textcircled{5} y = \frac{1}{x}$$

$$(3, \frac{1}{3})$$

solution

$$\frac{dy}{dx} = -1x^{-2}$$

$$\frac{dy}{dx} = -x^{-2}$$

$$\left. \frac{dy}{dx} \right|_{x=3} = -\frac{1}{3^2} = -\frac{1}{9}$$

$$m = \left. \frac{dy}{dx} \right|_{x=3} = -\frac{1}{9} = -\frac{1}{9} \quad m = -\frac{1}{9}$$

The equation of Tangent

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{3} = -\frac{1}{9}(x - 3)$$

$$y - \frac{1}{3} = -\frac{x-3}{9}$$

$$9y - 3 = -x + 3$$

$$9y + x - 6 = 0$$

$$9y + x - 6 = 0$$

Equation of the Normal

$$m_2 = -1 \times \frac{1}{m}$$

$$m_2 = 9$$

~

$$y - \frac{1}{3} = 9(x - 3)$$

$$y - \frac{1}{3} = 9x - 27$$

$$3y - 1 = 27x - 81$$

$$3y - 27x + 80 = 0$$

$$3y - 27x + 80 = 0$$