MATHS 107
Odeba Victoria Ehugocho
19/mf/501/280

Question
for the curves in problem 1 to 5, at the point given, find
(a) The equation of the tangent, (b) the equation of the normal

Solution
(1) $y=2x^2$ at the point $(1,2)$
$m=\frac{dy}{dx}=4x$

(a) $\left.\frac{dy}{dx}\right|_{x=1} = 4(1)=4$

$m=4, x_1=1, y_1=2.$
$y-y_1=m(x-x_1)$
$y-2=4(x-1)$
$y-2=4x-4$
$y-2-4x+4=0$
$y-4x+2=0$ {Equation of the tangent}

(b) Equation of the normal
$m_1m_2=-1$
$m_2=\frac{-1}{m_1}=\frac{-1}{4}=-0.25$

$y-y_1=m_2(x-x_1)$
$y-2=-\frac{1}{4}(x-1)$
$4y-8=-x+1$
$4y-8+x-1=0$
$4y+x-9=0$

(2) $y=3x^2-2x$ at the point $(2,8)$
$m=\frac{dy}{dx}=6x-2$

$\left.\frac{dy}{dx}\right|_{x=2} = 6(2)-2=12-2=10$

$m=10, x_1=2, y_1=8$

(a) $y - y_1 = m(x - x_1)$

$y - 8 = 10(x - 2)$

$y - 8 = 10x - 20$

$y - 8 - 10x + 20 = 0$

$y - 10x + 12 = 0$ {Equation of the tangent}

(b) $m_1 m_2 = -1$

$m_2 = \frac{-1}{m_1} = \frac{-1}{10}$

$y - y_1 = m(x - x_1)$

$y - 8 = \frac{-1}{10}(x - 2)$

$10y - 80 = -x + 2$

$10y - 80 + x - 2 = 0$

$10y + x - 82 = 0$ {Equation of the normal

5) $y = \frac{x^3}{2}$ $(-1, -\frac{1}{2})$

$m = \frac{dy}{dx} = 3 \cdot \frac{1}{2} = \frac{3}{2}x^2$

$\frac{dy}{dx}\Big|_{x=-1} = \frac{3}{2}(-1)^2 = \frac{3}{2}$

) $y - y_1 = m(x - x_1)$

$y - (-\frac{1}{2}) = \frac{3}{2}(x - (-1))$

$y + \frac{1}{2} = \frac{3}{2}(x + 1)$

$2y + 1 = 3x + 3$

$2y + 1 = 3x - 3 = 0$

$2y - 3x - 2 = 0$

$m_1 m_2 = -1$

$m_2 = \frac{-1}{m_1} = \cancel{-2} \frac{-1}{3/2} = -\frac{2}{3}$

$y - 8 - 10x + 20 = 0$

$y - 10x + 12 = 0$ {Equation of the tangent

(b) $m_1 m_2 = -1$

$m_2 = \frac{-1}{m_1} = \frac{-1}{10}$

$y - y_1 = m(x - x_1)$

$y - 8 = \frac{-1}{10}(x - 2)$

$10y - 80 = -x + 2$

$10y - 80 + x - 2 = 0$

$10y + x - 82 = 0$ {Equation of the normal

(3) $y = \frac{x^3}{2}$ $(-1, -\frac{1}{2})$

$m = \frac{dy}{dx} = 3 \cdot \frac{1}{2} = \frac{3}{2}x^2$

$\left.\frac{dy}{dx}\right|_{x=-1} = \frac{3}{2}(-1)^2 = \frac{3}{2}$

(a) $y - y_1 = m(x - x_1)$

$y - (-\frac{1}{2}) = \frac{3}{2}(x - (-1))$

$y + \frac{1}{2} = \frac{3}{2}(x + 1)$

$2y + 1 = 3x + 3$

$2y + 1 = 3x - 3 = 0$

$2y - 3x - 2 = 0$

(b) $m_1 m_2 = -1$

$m_2 = \frac{-1}{m_1} = \cancel{-2} \frac{-1}{3/2} = -\frac{2}{3}$

$y - y_1 = m_2(x - x_1)$

$y - -(-\frac{1}{2}) = -\frac{2}{3}(x - (-1))$

$3y + \frac{3}{2} = -2x - 2$

$3y + \frac{3}{2} + 2x + 2 = 0$

$3y + 2x + \frac{7}{2} = 0$

(1) $y = 2x^2$ at the point $(1, 2)$

$m = \frac{dy}{dx} = 4x$

(a) $\left.\frac{dy}{dx}\right|_{x=1} = 4(1) = 4$

$m = 4, x_1 = 1, y_1 = 2$

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y - 2 - 4x + 4 = 0$

$y - 4x + 2 = 0$ {Equation of the tangent}

(b) Equation of the normal

$m_1 m_2 = -1$

$m_2 = \frac{-1}{m_1} = \frac{-1}{4} = -0.25$

$y - y_1 = m_2(x - x_1)$

$y - 2 = -\frac{1}{4}(x - 1)$

$4y - 8 = -x + 1$

$4y - 8 + x - 1 = 0$

$4y + x - 9 = 0$

(2) $y = 3x^2 - 2x$ at the point $(2, 8)$

$m = \frac{dy}{dx} = 6x - 2$

$\left.\frac{dy}{dx}\right|_{x=2} = 6(2) - 2 = 12 - 2 = 10$

$m = 10,\ x_1 = 2,\ y_1 = 8$

dx

$m = \frac{dy}{dx}\Big|_{x=-2} = 1 - 2(-2) = 1 + 4 = 5$

$m = 5, y_1 = -5, x_1 = -2$

a) $y - y_1 = m(x - x_1)$

$y - (-5) = 5(x - (-2))$

$y + 5 = 5(x + 2)$

$y + 5 = 5x + 10$

$y + 5 - 5x - 10 = 0$

$y - 5x - 5 = 0$

(b) $m_1 m_2 = -1$

$m_2 = \frac{-1}{m_1} = \frac{-1}{5}$

$y - y_1 = m_2(x - x_1)$

$y + 5 = -\frac{1}{5}(x + 2)$

$5y + 25 = -x - 2$

$5y + 25 + x + 2 = 0$

$5y + x + 27 = 0$

(5) $y = \frac{1}{x} \quad (3, \frac{1}{3})$

$\frac{dy}{dx} = x^{-1} = -1 \cdot x^{-1-1} = -1x^{-2} = -x^{-2}$

$\frac{dy}{dx}\Big|_{x=3} \quad -x^{-2} = -(3)^{-2} = \frac{1}{-3^2} = \frac{1}{9}$

$m = \frac{1}{9}, \; x_1 = 3, \; y_1 = \frac{1}{3}$

a) $y - y_1 = m(x - x_1)$

$y - \frac{1}{3} = \frac{1}{9}(x - 3)$

$9y - 3 = x - 3$

$9y - 3 - x + 3 = 0$

$9y - x = 0$

4) $y = 1 + x - x^2 \qquad (-2, -5)$

$m = \frac{dy}{dx} = 1 - 2x$

$m = \frac{dy}{dx}\Big|_{x=-2} \qquad 1 - 2(-2) = 1 + 4 = 5$

$m = 5,\ y_1 = -5,\ x_1 = -2$

a) $y - y_1 = m(x - x_1)$

$y - (-5) = 5(x - (-2))$

$y + 5 = 5(x + 2)$

$y + 5 = 5x + 10$

$y + 5 - 5x - 10 = 0$

$y - 5x - 5 = 0$

b) $m_1 m_2 = -1$

$m_2 = \frac{-1}{m_1} = \frac{-1}{5}$

$y - y_1 = m_2(x - x_1)$

$y + 5 = -\frac{1}{5}(x + 2)$

$5y + 25 = -x - 2$

$5y + 25 + x + 2 = 0$

$5y + x + 27 = 0$

5) $y = \frac{1}{x} \qquad (3, \frac{1}{3})$

$\frac{dy}{dx} = x^{-1} = -1 \cdot x^{-1-1} = -1x^{-2} = -x^{-2}$

$\frac{dy}{dx}\Big|_{x=3} \qquad -x^{-2} = -(3)^{-2} = \frac{1}{-3^2} = \frac{-1}{9}$

$m = \frac{1}{9},\ x_1 = 3,\ y_1 = \frac{1}{3}$

a) $y - y_1 = m(x - x_1)$

$y - \frac{1}{3} = \frac{1}{9}(x - 3)$

b) $m_2 = \frac{-1}{m_1} = \frac{-1}{1/9} = -9$

$y - 1/3 = -9(x-3)$

$y - 1/3 = -9x + 27$

$y - 1/3 + 9x - 27 = 0$

$y + 9x - \frac{82}{3} = 0$

$3y + 27x - 82 = 0$