

5.) $y = 1/x$ at the point $(3, 1/3)$

$$m = \frac{dy}{dx} = -x^{-2} = -\frac{1}{x^2}$$

When $x = 3$, $m = -1/9$

a) Equation of a tangent: $y - y_1 = m(x - x_1)$

$$y - 1/3 = -1/9(x - 3)$$

$$9y - 3 = -1(x - 3)$$

$$9y - 3 = -x + 3$$

$= 9y + x - 6 = 0$, which is the equation of a tangent.

b) Equation of a normal: $y - y_1 = -1/m(x - x_1)$

$$y - 1/3 = 9(x - 3)$$

multiply all through by 3

$$3y - 1 = 27(x - 3)$$

$$3y - 1 = 27x - 81$$

$= 3y - 27x + 80 = 0$, which is the equation of the normal.

Assignment

For the curves in problem 1 to 5, at the points given, find

a) the equation of the tangent

b) the equation of the normal.

1) $y = 2x^2$ at the point $(1, 2)$

$$x = 1, y = 2(1)^2 = 2.$$

$$m = \frac{dy}{dx} = 4x = 4$$

a) Equation of tangent; $y - y_1 = m(x - x_1)$

$$y - 2 = 4(x - 1)$$

$$\Rightarrow y - 2 = 4x - 4$$

$$\Rightarrow y - 4x + 2 = 0, \text{ which is the equation of the tangent}$$

b) Equation of the normal, $y - y_1 = -\frac{1}{m}(x - x_1)$

$$y - 2 = -\frac{1}{4}(x - 1)$$

$$\Rightarrow -4y + 8 = x - 1$$

$$\Rightarrow -4y - x + 9 = 0$$

$$\Rightarrow 4y + x - 9 = 0, \text{ which is the equation of the normal.}$$

2) $y = 3x^2 - 2x$ at the point $(2, 8)$

$$m = \frac{dy}{dx} = 6x - 2$$

$$\text{when } x = 2, m = 10$$

a) Equation of tangent; $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 8 = 10(x - 2)$$

$$\Rightarrow y - 8 = 10x - 20$$

$$\Rightarrow y - 10x + 12 = 0, \text{ which is the equation of the tangent.}$$

b) Equation of the normal; $y - y_1 = -\frac{1}{m}(x - x_1)$

$$y - 8 = -\frac{1}{10}(x - 2)$$

$$\Rightarrow -10y + 80 = x - 2$$

$$\Rightarrow -10y - x + 82 = 0$$

$$\Rightarrow 10y + x - 82 = 0, \text{ which is the equation of a normal}$$

3) $y = x^3/2$ at the point $(-1, -1/2)$.

$$m = dy/dx$$

$$\text{Using quotient rule; } \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}, \quad \frac{dy}{dx} = \frac{2(3x^2) - x^3(0)}{2^2} = \frac{3x^2}{2}$$

$$\therefore \frac{dy}{dx} = \frac{3}{2}x^2$$

$$m = \frac{3}{2}(-1)^2 = \frac{3}{2}$$

a) Equation of a tangent: $(y - y_1) = m(x - x_1)$

$$y + 1/2 = \frac{3}{2}(x + 1)$$

$$\Rightarrow 2y + 1 = 3x + 3$$

$$\Rightarrow 2y - 3x - 2 = 0, \text{ which is the equation of a tangent.}$$

b) Equation of normal: $y - y_1 = -1/m(x - x_1)$

$$y + 1/2 = -\frac{2}{3}(x + 1)$$

Multiply all through by 6.

$$\Rightarrow 6y + 3 = -4(x + 1)$$

$$\Rightarrow 6y + 3 = -4x - 4$$

$$\Rightarrow 6y + 4x + 7 = 0, \text{ which is the equation of the normal.}$$

4) $y = 1 + x - x^2$ at the point $(-2, -3)$.

$$m = \frac{dy}{dx} = 1 - 2x$$

$$\text{when } x = -2, m = 1 - 2(-2) = 5$$

a) Equation of a tangent: $(y - y_1) = m(x - x_1)$

$$y + 3 = 5(x + 2)$$

$$y + 3 = 5x + 10$$

$$\Rightarrow y - 5x - 7 = 0, \text{ which is the equation of the tangent.}$$

Equation of a normal: $y - y_1 = -1/m(x - x_1)$

$$y + 3 = -1/5(x + 2)$$

$$-5y - 15 = x + 2$$

$$-5y - x - 17 = 0$$

$$\Rightarrow 5y + x + 17 = 0, \text{ which is the equation of the normal.}$$