Maths 104 (ASSIGNMENT 7)

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Matric Number: 19/MHS01/406
Serial Number: 008

1) $y = 2x^2$ at the point $(1,2)$

(i) Equation of the tangent

$m = 4x = 4(1) = 4$

$m = 4,\ x_1 = 1,\ y_1 = 2$

equation of tangent $\Rightarrow y - y_1 = m(x - x_1)$

$y - 2 = 4(x-1)$

$y - 2 = 4x - 4$

$y - 4x + 4 - 2 = 0$, $y - 4x + 2 = 0$

$\therefore$ Equation of the tangent $\Rightarrow y - 4x + 2 = 0$

(ii) Equation of the normal

For normal, $m_1 \times m_2 = -1$, $m_1 = 4$

$\frac{4}{4} \times m_2 = \frac{-1}{4}$, $m_2 = -1/4$

$m = -1/4,\ x_1 = 1,\ y_1 = 2$

equation of normal $\Rightarrow y - y_1 = m(x - x_1)$

$y - 2 = \frac{-1}{4}(x-1)$, $4(y-2) = -1(x-1)$

$4y - 8 = -x + 1$, $4y + x - 8 - 1 = 0$, $4y + x - 9 \Rightarrow 0$

$\therefore$ Equation of the normal $\Rightarrow 4y + x - 9 = 0$

2) $y = 3x^2 - 2x$ at the point $(2,8)$

(i) Equation of the tangent

$m = 6x - 2$, $6(2) - 2 \Rightarrow 12 - 2 \Rightarrow 10$, $\therefore m = 10$

$m = 10,\ y_1 = 8,\ x_1 = 2$

equation of tangent $\Rightarrow y - y_1 = m(x - x_1)$

$y - 8 = 10(x-2)$, $y - 8 = 10x + 20$, $y - 10x - 8 + 20 = 0$

$\therefore$ Equation of the tangent $\Rightarrow y - 10x + 12 = 0$

(ii) Equation of the normal

For normal, $m_1 \times m_2 = -1$, $m_1 = 10$

$\frac{10 \times m_2}{10} = \frac{-1}{10}$, $m_2 = -1/10$

$m = -1/10$, $x_1 = 2$, $y_1 = 8$

equation of normal $= y - y_1 = m(x - x_1)$

$y - 8 = \frac{-1}{10}(x - 2)$, $10(y - 8) = -x + 2$

$10y - 80 = -x + 2$, $10y + x - 82 = 0$

∴ Equation of the normal $\Rightarrow 10y + x - 82 = 0$

3) $y = x^3/2$ at the point $(-1, -1/2)$

(i) Equation of the tangent

$m = y = x^3/2$, $dy/dx = 1/2x^3$, $= 3/2x^2$

$\therefore m = 3/2x^2 = 3/2(-1)^2 = 3/2 \times 1 = 3/2$

$\therefore m = 3/2$, $x_1 = -1$, $y_1 = -1/2$

Equation of tangent $\Rightarrow y - y_1 = m(x - x_1)$

$y - -1/2 = \frac{3}{2}(x - -1)$, $y + \frac{1}{2} = \frac{3}{2}(x + 1)$ $= \frac{2y+1}{2} = \frac{3x+3}{2}$

$2(3x + 3) = 2(2y + 1)$, $6x + 6 = 4y + 2$

$6x - 4y + 4 = 0$

∴ Equation of the tangent $\Rightarrow 6x - 4y + 4 = 0$

(ii) Equation of the normal

For normal, $m_1 m_2 = -1$, $m_1 = 3/2$, $\frac{3}{2} \times m_2 = \frac{-1}{3/2}$, $m_2 = -1 \div \frac{3}{2}$

$m_2 = -1 \times \frac{2}{3}$, $m_2 = -\frac{2}{3}$

$m = -2/3$, $x_1 = -1$, $y_1 = -1/2$

equation of normal $\Rightarrow y - y_1 = m(x - x_1)$

$y - -1/2 = -\frac{2}{3}(x - -1)$, $y + 1/2 = -\frac{2}{3}(x + 1)$, $\frac{2y+1}{2} = -\frac{2x-2}{3}$

$2(-2x - 2) = 3(2y + 1)$, $-4x - 4 = 6y + 3$

$6y + 4x + 7 = 0$

∴ Equation of the normal $\Rightarrow 6y + 4x + 7 = 0$

4) $y = 1 + x - x^2$ at the point $(-2, -5)$.

(i) Equation of the tangent

$m = 1 - 2x \Rightarrow 1 - 2(-2) = 1 + 4 = 5$

$m = 5,\ x_1 = -2,\ y_1 = -5$

Equation of tangent $\Rightarrow y - y_1 = m(x - x_1)$

$y - -5 = 5(x - -2),\ y + 5 = 5(x + 2),\ y + 5 = 5x + 10$

$y - 5x - 5 = 0$

$\therefore$ Equation of the tangent $\Rightarrow y - 5x - 5 = 0$

(ii) Equation of the normal

for normal, $m_1 m_2 = -1,\ m_1 = 5$

$\frac{8 \times m_2}{8} = \frac{-1}{5},\ m_2 = -1/5$

$m_2 = -1/5,\ x_1 = -2,\ y_1 = -5$

Equation of normal $\Rightarrow y - y_1 = m(x - x_1)$

$y - -5 = -\frac{1}{5}(x - -2),\ y + 5 = \frac{-1}{5}(x + 2),\ 5(y + 5) = -x - 2$

$5y + 25 = -x - 2,\ 5y + x + 27 = 0$

$\therefore$ Equation of normal $\Rightarrow 5y + x + 27 = 0$

5) $y = 1/x$ at the point $(3, 1/3)$

$y = 1/x = x^{-1},\ m = -x^{-2}$

$m = -(3)^{-2} = -1/9$

$m = -1/9,\ x_1 = 3,\ y_1 = 1/3$

(i) Equation of tangent $\Rightarrow y - y_1 = m(x - x_1)$

$y - 1/3 = -\frac{1}{9}(x - 3),\ \frac{3y - 1}{3} = \frac{-x + 3}{9}$

$3(-x + 3) = 9(3y - 1),\ -3x + 9 = 27y - 9$

$\therefore 27y + 3x - 18 = 0 \Rightarrow$ Equation of tangent.

(ii) Equation of normal

For normal, $m_1 m_2 = -1,\ m_1 = -1/9$

$\frac{-1/9 \times m_2}{-1/9} = \frac{-1}{-1/9},\ m_2 = -1 \div \frac{-1}{9},\ \frac{-1}{1} \times \frac{9}{1},\ m = 9$

$m = 9,\ x_1 = 3,\ y_1 = 1/3$

Equation of normal $\Rightarrow y - y_1 = m(x - x_1)$

$y - 1/3 = 9(x-3)$, $\frac{3y-1}{3} \neq 9x-27$

$3y-1 = 3(9x-27)$

$3y-1 = 27x-81$, $3y-27x+80=0$

∴ Equation of normal $\Rightarrow 3y-27x+80 \Rightarrow 0$