Assignment VIII

1) Find the equation of the tangent and the normal at the points and for the curves given below:

1.) $y = 2x^2$ at the point (1, 2)

Solution: $x_1 = 1$, $y_1 = 2$

$$\frac{dy}{dx} = 4x$$

$$m = \frac{dy}{dx}\Big|_{x = x_1 = 1} \quad \therefore m = 4(1) = 4.$$

To find the equation of the tangent,

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 4(x - 1)$$
$$y - 2 = 4x - 4$$
$$y - 4x = -4 + 2$$
$$y - 4x = -2$$
$$y - 4x + 2 = 0$$

To find the equation of the normal,

$$m_1 m_2 = -1$$
$$4m_2 = -1$$
$$m_2 = -\frac{1}{4}$$
$$y - y_1 = m(x - x_1)$$
$$y - (2) = -\frac{1}{4}(x - 1)$$
$$4(y - 2) = -1(x - 1)$$
$$4y - 8 = -x + 1$$
$$4y + x = 1 + 8$$
$$4y + x = 9$$
$$4y + x - 9 = 0$$

2) $y = 3x^2 - 2x$ at point (2, 8).

Solution: $x_1 = 2$, $y_1 = 8$

$$\frac{dy}{dx} = 6x - 2$$

$$m = \frac{dy}{dx}\Big|_{x = x_1 = 2} \quad \therefore m = 6(2) - 2$$
$$m = 12 - 2$$
$$m = 10$$

To find the equation of the tangent,

$$y - y_1 = m(x - x_1)$$
$$y - 8 = 10(x - 2)$$
$$y - 8 = 10x - 20$$
$$y - 10x = -20 + 8$$
$$y - 10x = -12$$
$$y - 10x + 12 = 0$$

$= 2$ at the point $(-2, 5)$.

To find the equation of the normal,

$m_1 m_2 = -1$

$10 m_2 = -1$

$m_2 = \frac{-1}{10}$

$y - y_1 = m(x - x_1)$.

$y - 8 = -\frac{1}{10}(x - 2)$.

$10(y - 8) = -1(x - 2)$

$10y - 80 = -x + 2$

$10y + x = 2 + 80$

$10y + x = 82$

$10y + x - 82 = 0$.

3) $y = \frac{x^3}{2}$ at the point $(-1, -\frac{1}{2})$.

Solution: $x_1 = -1$, $y_1 = -\frac{1}{2}$

$\frac{dy}{dx} = \frac{3x^2}{2}$

$m = \frac{dy}{dx}\Big|_{x = x_1 = -1}$

$m = \frac{3(-1)^2}{2} = \frac{3 \times 1}{2} = \frac{3}{2}$ or $\frac{11}{2}$.

To find the equation of the tangent,

$y - y_1 = m(x - x_1)$

$y - (-\frac{1}{2}) = \frac{3}{2}(x - (-1))$.

$y + \frac{1}{2} = \frac{3}{2}(x + 1)$.

$2(y + \frac{1}{2}) = 3(x + 1)$

$2y + 1 = 3x + 3$

$2y - 3x = 3 - 1$

$2y - 3x = 2$

$2y - 3x - 2 = 0$.

To find the equation of the normal,

$m_1 m_2 = -1$

$\frac{3}{2} m_2 = -1$

$m_2 = -1 \times \frac{2}{3}$

$m_2 = -\frac{2}{3}$

$y - y_1 = m(x - x_1)$.

$y - (-\frac{1}{2}) = -\frac{2}{3}(x - (-1))$.

$y + \frac{1}{2} = -\frac{2}{3}(x + 1)$.

$3(y + \frac{1}{2}) = -2(x + 1)$.

$3y + \frac{3}{2} = -2x - 2$

$3y + 2x = -2 - \frac{3}{2}$

$3y + 2x = -\frac{7}{2}$

$2(3y + 2x) = -7$. or $3y + 2x + \frac{7}{2} = 0$

$6y + 4x + 7 = 0$.

5). $y = \frac{1}{x}$ at the point $(3, \frac{1}{3})$.

Solution: $x_1 = 3$, $y_1 = \frac{1}{3}$.

$y = x^{-1}$

$\frac{dy}{dx} = -x^{-2}$.

$m = \frac{dy}{dx}\Big|_{x = x_1 = 3}$.

$m = -3^{-2}$.

$m = -\frac{1}{9}$

$m = -\frac{1}{9}$

To find the equation of the tangent,

$y - y_1 = m(x - x_1)$

$y - \frac{1}{3} = -\frac{1}{9}(x - 3)$

$9(y - \frac{1}{3}) = -1(x - 3)$

$9y - 3 = -x + 3$

$9y + x = 3 + 3$

$9y + x = 6$

$9y + x - 6 = 0$

To find the equation of the normal,

$m_1 m_2 = -1$

$-\frac{1}{9} m_2 = -1$

$m_2 = -1 \times -\frac{9}{1}$

$m_2 = 9$

$y - y_1 = m(x - x_1)$.

$y - y_1 = 9(x - x_1)$

$y - \frac{1}{3} = 9(x - 3)$.

$y - \frac{1}{3} = 9x - 27$.

$y - 9x = -27 + \frac{1}{3}$.

$y - 9x = -\frac{80}{3}$

$3(y - 9x) = -80$.

$3y - 27x + 80 = 0$ or $y - 9x + \frac{80}{3} = 0$.

4) $y = 1 + x - x^2$ at the point $(-2, -5)$.

Solution: $x_1 = -2$, $y_1 = -5$.

$$\frac{dy}{dx} = 1 - 2x$$

$$m = \frac{dy}{dx}\Big|_{x = x_1 = -2}$$

$m = 1 - 2(-2)$

$m = 1 + 4$

$m = 5$

To find the equation of the tangent,

$y - y_1 = m(x - x_1)$

$y - (-5) = 5(x - (-2))$

$y + 5 = 5(x + 2)$

$y + 5 = 5x + 10$

$y - 5x = 10 - 5$

$y - 5x = 5$

$y - 5x - 5 = 0$

To find the equation of the normal,

$m_1 m_2 = -1$

$5 m_2 = -1$

$m_2 = \frac{-1}{5}$

$y - y_1 = m(x - x_1)$

$y - (-5) = -\frac{1}{5}(x - (-2))$

$(y + 5) = -\frac{1}{5}(x + 2)$

$5(y + 5) = -1(x + 2)$

$5y + 25 = -x - 2$

$5y + x = -2 - 25$

$5y + x = -27$

$5y + x + 27 = 0$