5. $y = \frac{1}{x}$ at the point $(3, \frac{1}{3})$

For normal

$y = \frac{1}{x}$ $\quad y = x^{-1}$

$\frac{dy}{dx} = -x^{-2}$

$m = \frac{dy}{dx} / x = 3$

$= -(3)^{-2}$

$-\frac{1}{3^2} = -\frac{1}{9}$

$m = -\frac{1}{9}$

The equation of the tangent

$y - y_1 = m(x - x_1)$

$(y - \frac{1}{3}) = \frac{1}{9}(x - 3)$

$9(y - \frac{1}{3}) = -1(x - 3)$

$9y - 3 = -x + 3$

$9y - x - 3 + 3 = 0$

$9y - x - 6 = 0$

The equation of the normal

$y - y_1 = -\frac{1}{m}(x - x_1)$

$(y - \frac{1}{3}) = -\frac{1}{\frac{1}{9}}(x - 3)$

$(y - \frac{1}{3}) = -\frac{1}{1} \div \frac{1}{9}(x - 3)$

$(y - \frac{1}{3}) = -\frac{1}{1} \times \frac{9}{1}(x - 3)$

$(y - \frac{1}{3}) = -9(x - 3)$

multiply both sides by 3

$3y - 1 = -27(x - 3)$

$3y - 1 = -27x - 81$

$3y + 27x - 1 + 81 = 0$

Collect like terms

$3y + 27x + 80 = 0$

For the curves in problem 1 to 3, at the points given find
(a) The equation of the tangent
(b) The equation of the normal

Solution

1. $y = 2x^2$ at the point $(1, 2)$

For the normal

$y = 2x^2$

$\frac{dy}{dx} = 4x$

$m = \frac{dy}{dx}(x=1)$

$m = 4(1)$

$m = 4$

Equation of the tangent

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y - 4x - 2 + 4 = 0$

$y - 4x + 2 = 0$

Equation of the normal

$m_1 m_2 = -1$

$y - y_1$

$m_2 = \frac{-1}{m_1}$

$y - y_1 = -1/m\,(x - x_1)$

$y - 2 = -1/4\,(x - 1)$

$4(y - 2) = -1(x - 1)$

$4y - 8 = -x + 1$

$4y + x - 8 - 1 = 0$

$4y + x - 9 = 0$

2. $y = 3x^2 - 2x$ at point $(2, 8)$

$\frac{dy}{dx} = 6x - 2$

$\frac{dy}{dx}$ at $x(2)$ $6(2) - 2$

$\frac{dy}{dx} = m$

Equation of Normal
$y - y_1 = m(x - x_1)$

2. $8x^2 - 2x$ at the point (2,8)

for normal

$y = 8x^2 - 2x$

$dy/dx = 6x - 2$

$m = dy/dx / x = 2$

$6(2) - 2$

$m = 12 - 2$

$\therefore m = 10$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y - 8 = 10(x - 2)$

$y - 8 = 10x - 20$

clt

$y - 10x - 8 + 20 = 0$

$y - 10x + 12 = 0$

Equation of the normal

$y - y_1 = -1/m(x - x_1)$

$y - 8 = -1/10(x - 2)$

$10(y - 8) = -1(x - 2)$

$10y - 80 = -x + 2$

clt

$10y + x - 80 - 2 = 0$

$10y + x - 82 = 0$

$3(y+2) = -1(x+1)$

$3y+1.5 = -x-1$

$3y+x+1.5+1=0$

$3y+x+2.5=0$

4 $y = 1+x-x^2$ at the point $(-2,-5)$

$y = 1+x-x^2$

$dy/dx = 1-2x$

$\frac{dy}{dx}$ at $x=-2$ $\quad 1-2(-2)$

$m = \frac{dy}{dx} = 1+4$

$m = 5$

Equation of tangent

$y-y_1 = 5(x-x_1)$

$(y+5) = 5(x+2)$

$(y+5) = 5x+10$

$y-5x+5-10=0$

$y-5x-5=0$

Equation of the normal

$y-y_1 = -1/m(x-x_1)$

$(y-(-5)) = -1/5(x-(-2))$

$(y+5) = -1/5(x+2)$

$(y+5) = -1/5(x+2)$

$5(y+5) = -1(x+2)$

$5y+25 = -x-2$

$5y+x+25+2=0$

$5y+x+27=0$

3) For equation of normal $y = \frac{1}{2}x^{3/2}$ at the point $(-1, -\frac{1}{2})$

For normal

$y = x^{3/2}$

$\frac{dy}{dx}$ of $\frac{1}{2}x^3$

$\frac{dy}{dx} = \frac{3}{2}x^2$

$m = \frac{dy}{dx}|_{x=1} \quad \frac{3}{2}(1)^2 \quad \therefore m = \frac{3}{2}$

The equation of tangent

$y - y_1 = m(x - x_1)$

$(y - (-\frac{1}{2})) = \frac{3}{2}(x - (-1))$

$(y + \frac{1}{2}) = \frac{3}{2}(x + 1)$

$2(y + \frac{1}{2}) = 3(x + 1)$

$2y + 1 = 3x + 3$

clt

$2y - 3x + 1 - 3 = 0$

$2y - 3x - 2 = 0$

The equation of normal

$y - y_1 = -\frac{1}{m}(x - x_1)$

$(y - (-\frac{1}{2})) = -\frac{1}{\frac{3}{2}}(x - (-1))$

$(y + \frac{1}{2}) = -\frac{2}{3}(x + 1)$

$3(y + \frac{1}{2}) = -2(x + 1)$

both side multiply by 6

$6(y + \frac{1}{2}) = -4(x + 1)$

$6y + 3 = -4x - 4$

clt

$6y + 4x + 3 + 4 = 0$

$6y + 4x + 7 = 0$