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COURSE CODE : MAT 104

SOLUTIONS TO QUESTIONS GIVEN

Find the (a) equation of tangent and (b) equation of the normal

i.) $Y = 2x^2$ at the point $(1, 2)$

ii.) $Y = 3x^2 - 2x$ at the point $(2, 8)$

iii.) $Y = \frac{x^3}{2}$ at the point $(-1, -\frac{1}{2})$

iv.) $y = 1 + x - x^2$ at the point $(-2, -5)$

v.) $y = \frac{1}{x}$ at the point $(3, \frac{1}{3})$

SOLUTION

1.) a) equation of tangent

$y = 2x^2$

differentiate : $y = 2x^2$

$\frac{dy}{dx} = 4x$

You need the slope to find the equation

∴ Slope of tangent at point $(1, 2)$ where $x = 1$

$M_{tangent} = 4(1) = 4$

∴ equation of tangent ; using $y - y_1 = m(x - x_1)$ where $x_1 = 1$ & $y_1 = 2$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$4x - y - 4 + 2 = 0$

$4x - y - 2 = 0$; equation of tangent.

Slope of normal = negative reciprocal of the slope of tangent

If $M_{tangent} = 4$

∴ $N_{normal} = -\frac{1}{4}$

∴ The equation of Normal ; Using $y - y_1 = m(x - x_1)$

where $x_1 = 1$ & $y_1 = 2$

$y - 2 = -\frac{1}{4}(x - 1)$

Cross multiply

$4(y - 2) = -1(x - 1)$, $4y - 8 = -x + 1$

$x + 4y - 8 - 1 = 0$

Equation of Normal $= x + 4y - 9 = 0$

2.) $y = 3x^2 - 2x$ at point $(2, 8)$

differentiate $\frac{dy}{dx} = 6x - 2$

at point $(2, 8)$ where $x = 2$

Slope of Tangent $= 6(2) - 2 = 12 - 2 = 10$

$= 10$

slope of Normal $= -\frac{1}{10}$ (negative reciprocal of $M_{Tangent}$)

a) equation of Tangent ; $y - y_1 = m(x - x_1)$ $x_1 = 2,\ y_1 = 8$

$y - 8 = 10(x - 2)$, $y - 8 = 10x - 20$

$10x - y - 20 + 8 = 0$ $10x - y - 12 = 0$

∴ The equation of tangent $= 10x - y - 12 = 0$

b) equation of Normal

$y - 8 = \frac{-1}{10}(x - 2)$

Cross multiply

$10(y - 8) = -1(x - 2)$

$10y - 80 = -x + 2$

$x + 10y - 80 - 2 = 0$

$x + 10y - 82 = 0$; Equation of Normal

$Y = \frac{x^3}{2}$ at point $(-1, -\frac{1}{2})$

$Y = \frac{x^3}{2}$ $\begin{matrix} u \\ v \end{matrix}$ $u = x^3$ $\frac{du}{dx} = 3x^2$ using quotient rule

$v = 2$ $\frac{dv}{dx} = 0$ $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{2(3x^2) - x^3(0)}{4}$

$\frac{6x^2 - 0}{4} = \frac{6x^2}{4} = \frac{3x^2}{2}$

where $x = -1$

$= \frac{3x^2}{2} = \frac{3(-1)^2}{2} = \frac{3}{2}$

$-\frac{1}{3/2} = -\frac{2}{3}$

of tangent ; $y - y_1 = m(x - x_1)$ where $x = -1$, $y = -\frac{1}{2}$

$= \frac{3}{2}(x - (-1))$, $\frac{y}{1} + \frac{1}{2} = \frac{3}{2}(x + 2)$

$\frac{3}{2}(x + 2)$, $2(2y + 1) = 6(x + 2)$

$= 6x + 12$, $6x - 4y + 12 - 2 = 6x - 4y + 10 = 0$

of tangent ; $6x - 4y + 10 = 0$

of normal

$= -\frac{2}{3}(x - (-1))$, $\frac{y}{1} + \frac{1}{2} = -\frac{2}{3}(x + 1)$

$-\frac{2}{3}(x + 1)$, $3(2y + 1) = -4(x + 1)$

$= -4x - 4$

$y + 3 + 4 = 0$

$+ 12 = 0$; Equation of Normal

$+ x - x^2$ at point $(-2, -5)$

$+ 1 - 2x$ $\frac{dy}{dx} = 1 - 2x$

where $x = -2$

$1 - 2(-2) = 1 + 4$

$= 5 =$ slope of tangent

$= -\frac{1}{5} =$ slope of Normal

tangent ; $y - y_1 = m(x - x_1)$ where $x_1 = -2$ & $y_1 = -5$

$-5) = 5(x - (-2))$

$y + 5 = 5x + 10$

$5x - y + 10 - 5 = 0$

$5x - y + 5 = 0$; Equation of tangent

b) Equation of Normal

$y - (-5) = -\frac{1}{5}(x - (-2))$

$y + 5 = -\frac{1}{5}(x + 2)$, $5(y + 5) = -1(x + 2)$

$5y + 25 = -x - 2$, $x + 5y + 25 + 2 = 0$

$x + 5y + 27 = 0$; Equation of Normal

5) $y = \frac{1}{x}$ Point $(3, \frac{1}{3})$

$\frac{dy}{dx} = x^{-1} = -x^{-1-1} = -x^{-2}$

where $x = 3$

$M_{tangent} = -(3)^{-2}$

$= -\frac{1}{9}$

$M_{normal} = -\left(-\frac{1}{1/9}\right) = \frac{9}{1} = 9$

$M_{normal} = 9$

a) equation of tangent $= y - y_1 = m(x - x_1)$ where $x_1 = 3$, $y_1 = \frac{1}{3}$

$y - (\frac{1}{3}) = -\frac{1}{9}(x - 3)$, $\frac{y}{1} - \frac{1}{3} = -\frac{1}{9}(x - 3)$

$\frac{3y - 1}{3} \times -\frac{1}{9}(x - 3)$, $9(3y - 1) = -3(x - 3)$

$18y - 9 = -3x + 9$, $18y + 3x - 9 - 9 = 0$

Equation of tangent ; $3x + 18y - 18 = 0$

b) equation of Normal

$\frac{y}{1} - \frac{1}{3} = 9(x - 3)$

$\frac{3y - 1}{3} \times \frac{9(x - 3)}{1}$

$3y - 1 = 18(x - 3)$ $3y - 1 = 18x - 54$

$18x - 3y - 54 + 1 = 0$

$18x - 3y - 53 = 0$; Equation of Normal.