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QUESTION AND ANSWER

For the curves in Problem 1 to 5, at the points given, find
a) the equation of the tangent
b) the equation of the normal.

1. $y = 2x^2$ at the point $(1, 2)$

Solution

a. For equation of the tangent $[x_1 = 1, y_2 = 2]$

$$\frac{dy}{dx} = 4x$$

$$m_1 = \left.\frac{dy}{dx}\right|_{x = x_1}$$

$$m_1 = 4(1) = 4$$

$$y - y_1 = m_1(x - x_1)$$

$$y - 2 = 4(x - 1)$$

$$y - 2 = 4x - 4$$

$\therefore y - 4x + 2 = 0$ is the equation of the tangent.

b. For equation of the normal

$$m_2 = \frac{-1}{m_1} = \frac{-1}{4}$$

$$y - y_1 = \frac{-1}{m_1}(x - x_1)$$

$$y - 2 = \frac{-1}{4}(x - 1)$$

$$\frac{y - 2}{1} = \frac{-1}{4}(x - 1)$$

$$4(y - 2) = -1(x - 1)$$

$$4y - 8 = -x + 1$$

$\therefore$ $4y + x - 9 = 0$ is the equation of the normal.

2. $y = 3x^2 - 2x$ at the point $(2, 8)$

Solution

a For the equation of the tangent: $[x_1 = 2, y_1 = 8]$

$$\frac{dy}{dx} = 6x - 2$$

$$m_1 = \left.\frac{dy}{dx}\right|_{x = x_1}$$

$$m_1 = 6(2) - 2$$

$$m_1 = 12 - 2$$

$$\therefore m_1 = 10$$

$$y - y_1 = m(x - x_1)$$

$$y - 8 = 10(x - 2)$$

$$y - 8 = 10x - 20$$

$\therefore$ $y - 10x + 12 = 0$ is the equation of the tangent.

b. For the equation of the normal:

$$m_2 = \frac{-1}{m_1} = \frac{-1}{10}$$

$$y - y_1 = \frac{-1}{m_1}(x - x_1)$$

$$y - 8 = \frac{-1}{10}(x - 2)$$

$$\frac{y - 8}{1} = \frac{-1}{10}(x - 2)$$

$$10(y - 8) = -1(x - 2)$$

$$10y - 80 = -x + 2$$

$\therefore$ $10y + x - 82 = 0$ is the equation of the normal

3) $y = \dfrac{x^3}{2}$ at the point $(-1, -1/2)$

Solution

$[x_1 = -1,\ y_1 = -1/2]$

Using quotient rule

$$\frac{dy}{dx} = \frac{V\,du/dx - U\,dv/dx}{V^2}$$

Let $v = 2$ and $u = x^3$

$du/dx = 3x^2, \quad dv/dx = 0$

$$\frac{dy}{dx} = \frac{2 \cdot 3x^2 - x^3 \cdot 0}{2^2}$$

$$\frac{dy}{dx} = \frac{6x^2 - 0}{4}$$

$$\therefore \frac{dy}{dx} = \frac{6x^2}{4}$$

a. For the equation of the tangent

$$m_1 = \left.\frac{dy}{dx}\right|_{x = x_1}$$

$$m_1 = \left.\frac{dy}{dx}\right|_{x = -1}$$

$$m_1 = \frac{6(-1)^2}{4}$$

$$m_1 = \frac{6}{4} = \frac{3}{2}$$

$$y - y_1 = m_1(x - x_1)$$

$$y + \frac{1}{2} = \frac{3}{2}(x + 1)$$ [Multiply through by 2]

$$2y + 1 = 3(x + 1)$$

$$2y + 3x = 10$$

$9y + x - 3 - 3 = 0$

$9y + x - 6 = 0$

$\therefore$ $9y + x - 6 = 0$ is the equation of the tangent.

b. For the equation of the normal.

$$m_2 = \frac{-1}{m_1} = \frac{-1}{-1/9}$$

$$m_2 = -\cancel{1} \times \frac{9}{-\cancel{1}}$$

$$m_2 = 9$$

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - y_1 = m_2(x - x_1)$$

$$y - \frac{1}{3} = 9(x - 3)$$

$$y - \frac{1}{3} = 9x - 27$$

$$\frac{y}{1} - \frac{1}{3} = \frac{9x}{1} - \frac{27}{1}$$

Multiply through by 3

$$3\left(\frac{y}{1}\right) - \left[\cancel{3}\left(\frac{1}{\cancel{3}}\right)\right] = 3\left(\frac{9x}{1}\right) - \left[3\left(\frac{27}{1}\right)\right]$$

$$3y - 1 = 27x - 81$$

$$3y - 27x - 1 + 81 = 0$$

$$3y - 27x + 80 = 0$$

$\therefore$ $3y - 27x + 80 = 0$ is the equation of the normal.

$2y + 1 = 3x + 3$

$\therefore 2y - 3x - 2 = 0$ is the equation of the tangent.

b. For the equation of the normal.

$$m_2 = \frac{-1}{m_1} = \frac{-1}{3/2}$$

$$m_2 = -1 \times \frac{2}{3}$$

$$\therefore m_2 = \frac{-2}{3}$$

$$y - y_1 = \frac{-1}{m_1}(x - x_1)$$

$$y + \frac{1}{2} = -\frac{2}{3}(x + 1)$$

$$y + \frac{1}{2} = \frac{-2x - 2}{3}$$

$$y + \frac{1}{2} = \frac{-2x}{3} - \frac{2}{3}$$

Multiply through by 6

$$6(y) + \cancel{6}^{3}\left(\frac{1}{\cancel{2}_{1}}\right) = \cancel{6}^{2}\left(\frac{-2x}{\cancel{3}_{1}}\right) - \cancel{6}^{2}\left(\frac{2}{\cancel{3}_{1}}\right)$$

$$6y + 3 = 2(-2x) - 4$$

$$6y + 3 = -4x - 4$$

$$6y + 4x + 4 + 3 = 0$$

$$6y + 4x + 7 = 0$$

$\therefore 6y + 4x + 7 = 0$ is the equation of the normal.

4. $y = 1 + x - x^2$ at the point $(-2, -5)$

Solution

a) For the equation of the tangent. $[x_1 = -2, y_1 = -5]$

5 $y = \frac{1}{x}$ at the point $(3, 1/3)$

Solution

$$y = x^{-1}$$

$$[x_1 = 3, y_1 = 1/3]$$

$$\frac{dy}{dx} = -x^{-2}$$

a For the equation of the tangent.

$$m_1 = \frac{dy}{dx} \Big| x = x_1$$

$$m_1 = \frac{dy}{dx} \Big| x = 3$$

$$m_1 = -1(-3)^{-2}$$

$$m_1 = -\frac{1}{3^2}$$

$$\therefore m_1 = -\frac{1}{9}$$

$$y - y_1 = m_1(x - x_1)$$

$$y - \frac{1}{3} = \frac{-1}{9}(x - 3)$$

$$y - \frac{1}{3} = \frac{-1x + 3}{9}$$

$$y - \frac{1}{3} = \frac{-1}{9}x + \frac{3}{9}$$

Multiply through by 9

$$9(y) - \left[\cancel{9}^{3}\left(\frac{1}{\cancel{3}_{1}}\right)\right] = \cancel{9}^{1}\left(\frac{-1}{\cancel{9}_{1}}x\right) + \cancel{9}^{1}\left(\frac{3}{\cancel{9}_{1}}\right)$$

$$9y - 3 = -1x + 3$$

$$9y - 3 = -x + 3$$

$$\frac{dy}{dx} = -2x + 1$$

$$m_1 = \frac{dy}{dx} \Big|_{x - x_1}$$

$m_1 = -2(-2) + 1 \; \cancel{=5}$

$m_1 = 4 + 1$

$\therefore m_1 = 5$

$y - y_1 = m_1(x - x_1)$

$y + 5 = 5(x + 2)$

$y + 5 = 5x + 10$

$y - 5x + 5 - 10 = 0$

$y - 5x - 5 = 0$

$\therefore$ $y - 5x - 5 = 0$ is the equation of the tangent.

b. For the equation of the normal

$$m_2 = \frac{-1}{m_1} = \frac{-1}{5}$$

$$\therefore m_2 = \frac{-1}{5}$$

$y - y_1 = m_2(x - x_1)$

$$y + 5 = \frac{-1}{5}(x + 2)$$

$$\frac{y + 5}{1} = \frac{-1}{5}(x + 2)$$

$5(y + 5) = -1(x + 2)$

$5y + 25 = -x - 2$

$5y + x + 25 + 2 = 0$

$5y + x + 27 = 0$

$\therefore$ $5y + x + 27 = 0$ is the equation of the normal.