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COURSE: MAT 104

DEPT: MECHATRONICS ENGINEERING

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① Find the limit of the function $\frac{x - \cos x}{x}$ as $x \to 0$

Solution

$$\lim_{x \to 0} \frac{x - \cos x}{x}$$

$$= \lim_{x \to 0} 1 - \frac{\cos x}{x}$$

$$= 1 - \lim_{x \to 0} \frac{\cos x}{x}$$

Using L'Hopital's Rule

$$= 1 - \lim_{x \to 0} \frac{\frac{d \cos x}{dx}}{\frac{dx}{dx}}$$

$$= 1 - \lim_{x \to 0} \frac{-\sin x}{1}$$

$$= 1 - \lim_{x \to 0} -\sin x$$

$$= 1 + \sin 0$$

$$= 1 + 0$$

$$= 1$$

(12) If $y = -3\tan 7x \cdot e^{3x}$ find $\frac{dy}{dx}$

Soln

Let $u = -3$ $v = \tan 7x$ $w = e^{3x}$

$$\frac{dy}{dx} = uvw\left(\frac{1}{u}\cdot\frac{du}{dx} + \frac{1}{v}\cdot\frac{dv}{dx} + \frac{1}{w}\cdot\frac{dw}{dx}\right)$$

$$= -3\tan 7x \cdot e^{3x}\left(0 + \frac{7\sec^2 7x}{\tan 7x} + 3\right)$$

$$= -3\tan 7x \cdot e^{3x}\left(\frac{7\sec^2 7x}{\tan 7x} + 3\right)$$

$$= -3\tan 7x \cdot e^{3x}\left(7\operatorname{cosec} 7x \sec 7x + 3\right)$$

$$= -3\tan 7x \cdot e^{3x}\left(\frac{7\sec^2 7x}{\tan 7x} + 3\right)$$

$$\therefore \frac{dy}{dx} = -3\tan 7x \cdot e^{3x}\left(7\operatorname{cosec} 7x \sec 7x + 3\right)$$

OR $\frac{dy}{dx} = -3\tan 7x \cdot e^{3x}\left(7\operatorname{cosec} 7x \sec 7x + 3\right)$

(3) If $y = \cos 3x$ find $\frac{dy}{dx}$ using first principle

Soln

Let $f(x) = \cos 3x = y$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - fx}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(\frac{\cos 3(x+\Delta x) - \cos 3x}{\Delta x} \right)$$

$$= \lim_{\Delta x \to 0} \left[\frac{-2 \sin\left(\frac{3x+3\Delta x+3x}{2}\right) \sin\left(\frac{3x+3h-3x}{2}\right)}{\Delta x} \right]$$

from $\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$

$$= \lim_{\Delta x \to 0} \left(\frac{-2 \sin\left(\frac{6x+3\Delta x}{2}\right) \sin\left(\frac{3\Delta x}{2}\right)}{\Delta x} \right)$$

$$= -2 \lim_{\Delta x \to 0} \sin\left(\frac{6x+3\Delta x}{2}\right) \cdot \lim_{\Delta x \to 0} \frac{\sin \frac{3\Delta x}{2}}{\Delta x}$$

where $\lim_{x \to 0} \frac{\sin ax}{x} = a$

$$\frac{dy}{dx} = -2 \sin\left(\frac{6x+0}{2}\right) \cdot \frac{3}{2}$$

$$\frac{dy}{dx} = -3 \sin 3x$$

(4) Given that $f(x) = 2x^3 - 7x$ $g(x) = -3x$ $(f-g)(5)$

Soln

$(f-g)(x) = f(x) - g(x)$

$= 2x^3 - 7x - (-3x)$

$= 2x^3 - 7x + 3x$

$= 2x^3 - 4x$

$(f-g)(5) = 2(5)^3 - 4(5)$

$= 230$

(5) Find $fog(x)$ if $f(x) = 4x^2 + 2$ $g(x) = 2x + 3$

$fog(x) = 4(2x+3)^2 + 2$

$= 4(4x^2 + 12x + 9) + 2$

$= 4[4x^2 + 12x + 9] + 2 = 16x^2 + 48x + 38$

(6) Find the gradient of $x^2 + 2xy + y^2 = 1924$

Soln: $2x + 2y$

⑥ Find the gradient of $x^2 + 2xy + y^2 = 1020$

Soln

$$x^2 + 2xy + y^2 = 1020$$

$$2x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 1020$$

$$2x + 2y - 1020 = (-2x - 2y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x + 2y - 1020}{-2(x+y)}$$

$$= \frac{2(x+y-510)}{-2(x+y)}$$

$$= -\frac{(x+y-510)}{(x+y)} = -1$$

~~If $x^2 + 2xy + y^2 = 1020$ is~~

⑦ Find the first derivative of the fuction $y = x^2 \cos x$

Soln : $u = x^2$ $v = \cos x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{du}{dx} = 2x \; ; \; \frac{dv}{dx} = -\sin x$$

$$\frac{dy}{dx} = x^2(-\sin x) + 2x(\cos x)$$

$$\frac{dy}{dx} = 2x\cos x - x^2 \sin x$$