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MEDICINE AND SURGERY
19/MHS01/323
MAT 104 ASSIGNMENT

For the curves in problem 1 to 5, at the points given, find a) the equation of the tangent and b) the equation of the normal.

1) $y = 2x^2$ at the point (1, 2)

Solution.

$y = 2x^2$

$\frac{dy}{dx} = 4x$

$\left.\frac{dy}{dx}\right|_{x=1} = 4(1) = 4$

$m = 4$.

$x_1 = 1 \quad y_1 = 2$

✓ Equation of tangent

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y - 4x = -4 + 2$

$y - 4x + 2 = 0$

Tangent equation $\Rightarrow y - 4x + 2 = 0$ ①

✓ Equation of normal

$m_1 m_2 = -1$

$4 m_2 = -1$

$m_2 = \frac{-1}{4}$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-1}{4}(x - 1)$

$y-2 = \frac{-x+1}{4}$

$4y-8 = -x+1$

$4y+x = 1+8$

$4y+x = 9$

$4y+x-9=0$

Normal Equation $\Rightarrow$ $4y+x-9=0$

2) $y = 3x^2-2x$ at the point $(2,8)$

Solution.

$y = 3x^2-2x$

$\frac{dy}{dx} = 6x-2$

To determine the gradient

$\frac{dy}{dx}\Big|_{x=2} = 6(2)-2$

$12-2 = 10$

$M = 10$

Equation of tangent.

$y-y_1 = m(x-x_1)$

$y-8 = 10(x-2)$

$y-8 = 10x-20$

$y-10x = -20+8$

$y-10x = -12$

$y-10x+12 = 0$

Tangent Equation $\Rightarrow$ $y-10x+12=0$

Equation of normal

$y-y_1 = \frac{-1}{m_1}(x-x_1)$

$y-8 = \frac{-1}{10}(x-2)$

$y-8 = \frac{-x+2}{10}$

$10y - 80 = -x + 2$

$10y + x = 2 + 80$

$10y + x = 82$

$10y + x - 82 = 0$

Normal Equation $\Rightarrow 10y + x - 82 = 0$

3) $y = \frac{x^3}{2}$ at the point $(-1, -\frac{1}{2})$

Solution.

$y = \frac{x^3}{2} = \frac{1}{2}x^3$

$\frac{dy}{dx} = \frac{3}{2}x^2$

To determine the gradient

$\frac{dy}{dx}\Big|_{x=-1} = \frac{3}{2}(-1)^2 = \frac{3}{2}$

$m = \frac{3}{2}$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y + \frac{1}{2} = \frac{3}{2}(x + 1)$

$y + \frac{1}{2} = \frac{3}{2}x + \frac{3}{2}$

$y - \frac{3}{2}x = \frac{3}{2} - \frac{1}{2}$

$y - \frac{3}{2}x = 1$

$y - \frac{3}{2}x - 1 = 0$

Tangent Equation $\Rightarrow y - \frac{3}{2}x - 1 = 0$

Equation of Normal

$y - y_1 = m(x - x_1)$

$y + \frac{1}{2} = -\frac{2}{3}(x + 1)$

$y + \frac{1}{2} = \frac{-2x - 2}{3}$

$3y + \frac{3}{2} = -2x - 2$

$3y + 2x = -2 - \frac{3}{2}$

$3y + 2x = -\frac{7}{2}$

$3y + 2x + \frac{7}{2} = 0$

Normal Equation $\Longrightarrow 3y + 2x + \frac{7}{2} = 0$

4) $y = 1 + x - x^2$ at the point $(-2, -5)$

Solution.

$y = 1 + x - x^2$

$\frac{dy}{dx} = 1 - 2x$

To determine the gradient

$\frac{dy}{dx}\Big|_{x=-2} \quad 1 - 2(-2) = 1 + 4 = 5$

$m_1 = 5$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y + 5 = 5(x + 2)$

$y + 5 = 5x + 10$

$y - 5x = 10 - 5$

$y - 5x = 5$

$y - 5x - 5 = 0$

Tangent Equation $\Longrightarrow y-5x-5=0$

↳ Equation of Normal

$y-y_1 = m(x_0-x_1)$

$y+5 = \frac{-1}{5}(x+2)$

$y+5 = \frac{-x-2}{5}$

$5y+25 = -x-2$

$5y+x = -2-25$

$5y+x = -27$

$5y+x+27=0$

Normal Equation $\Longrightarrow 5y+x+27=0$

5) $y = \frac{1}{x}$ at the point $(3, \frac{1}{3})$

Solution

$y = \frac{1}{x} = x^{-1}$

$\frac{dy}{dx} = -x^{-2}$

To determine the gradient

$\frac{dy}{dx}\Big|_{x=3} (-3^{-2}) = \frac{-1}{3^2} = -\frac{1}{9}$

$m = -\frac{1}{9}$

↳ Equation of tangent

$y-y_1 = m(x-x_1)$

$y-\frac{1}{3} = \frac{-1}{9}(x-3)$

$y-\frac{1}{3} = \frac{-x+3}{9}$

$9y-3 = -x+3$

$9y + x = 3 + 3$

$9y + x = 6$

$9y + x - 6 = 0$

Tangent Equation $\Rightarrow$ $9y + x - 6 = 0$

Equation of normal

$y - y_1 = m(x - x_1)$ / gradient $m_2 = 9$

$y - \frac{1}{3} = 9(x - 3)$

$y - \frac{1}{3} = 9x - 27$

$y - 9x = -27 + \frac{1}{3}$

$y - 9x = \frac{-80}{3}$

$3y - 27x = -80$

$3y - 27x + 80 = 0$

Normal Equation $\Rightarrow$ $3y - 27x + 80 = 0$