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1. Find the limit of the function $\left(\frac{x-\cos x}{x}\right)$ as $x \to 0$

Solution

$$\lim_{x\to 0} \frac{x-\cos x}{x}$$

By direct substitution $x \to 0 : \frac{-1}{0}$, using L'hospital's Rule

$$\lim_{x\to 0} \frac{1-(-\sin x)}{1}$$

$$\lim_{x\to 0} \frac{1+\sin x}{1}$$

$$\lim_{x\to 0} \frac{1+\sin 0}{1}$$

$$= 1$$

2. If $y = -3\tan 7x e^{3x}$ find dy/dx

Solution

$$y = -3\tan 7x e^{3x}$$

$$u = -3 \quad v = \tan 7x \quad w = e^{3x}$$

$$\frac{du}{dx} = 0 \quad \frac{dv}{dx} = 7\sec^2 7x \quad \frac{dw}{dx} = 3e^{3x}$$

$$\frac{dy}{dx} = y\left[\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}\frac{dw}{dx}\right]$$

$$\frac{dy}{dx} = \frac{-3\tan 7x e^{3x}}{-3\tan 7x e^{3x}}\left[\frac{1}{-3}(0) + \frac{1}{\tan 7x}(7\sec^2 7x) + \frac{1}{e^{3x}}(3e^{3x})\right]$$

$$\frac{dy}{dx} = \frac{-3\tan 7x e^{3x}}{-3\tan 7x e^{3x}}\left[0 + \frac{7\sec^2 7x}{\tan 7x} + 3\right]$$

OR

$$\frac{dy}{dx} = -3\tan 7x e^{3x}\left[7\,\mathrm{cosec}\,7x \sec 7x + 3\right]$$

3 If $y = \cos 3x$. find dy/dx from the first principle

solution

$$y = \cos 3x$$

$$\Delta y = \cos 3(x + \Delta x) - y$$

$$\Delta y = \cos 3(x + \Delta x) - \cos 3x$$

where $\cos A + \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$.

$$A = x + \Delta x \qquad B = x$$

$$\frac{A+B}{2} = \frac{2x + \Delta x}{2} \qquad \frac{A-B}{2} = \frac{\Delta x}{2}$$

$$\Delta y = -2 \sin 3\left(\frac{2x + \Delta x}{2}\right) \sin 3\left(\frac{\Delta x}{2}\right)$$

$$\frac{\Delta y}{\Delta x} = \frac{-2 \sin 3\left(\frac{2x + \Delta x}{2}\right) \sin 3\left(\frac{\Delta x}{2}\right) \times \frac{1}{2}}{\Delta x \times \frac{1}{2}}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -\sin 3\left(\frac{2x + \Delta x}{2}\right) \frac{\left(\sin 3\frac{\Delta x}{2}\right)}{\frac{3\Delta x}{2}}$$

where $\frac{\sin ax}{x} = a$

$$\frac{dy}{dx} = -\sin 3\left(\frac{2x + 0}{2}\right) \cdot 3$$

$$\frac{dy}{dx} = -3 \sin 3x$$

4 Given that $f(x) = 2x^3 - 7x$ and $g(x) = -3x$. find $(f-g)(5)$

Solution

$$(f-g)(5)$$

$$(f-g)(x) = 2x^3 - 7x - (-3x)$$

$$2x^3 - 4x$$

$$(f-g)(5) = 2(5^3) + 4(5)$$

$(f \cdot g)(5) = -250 - 20$

$= 230$

5) find $fog(x)$ if $f(x) = 4x^2 + 2$ and $g(x) = 2x + 3$

solution

$fog(x) = 4(2x+3)^2 + 2$

$4[4x^2 + 12x + 9] + 2$

$16x^2 + 48x + 36 + 2$

$fog(x) = 16x^2 + 48x + 38$

6 find the gradient of $x^2 + 2xy + y^2 = 1,020$

Solution

$x^2 + 2xy + y^2 - 1020 = 0$

$\frac{dy}{dx} = m$

$\frac{dy}{dx}$ $2x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} - 0$

$2x + 2y = -\frac{dy}{dx}[2x + 2y]$

$\frac{dy}{dx} = \frac{2x + 2y}{-(2x+2y)}$

$\frac{dy}{dx} = -1$

∴ The gradient = -1

Find the first derivative of the function $y = x^2 \cos x$

solution

$y = x^2 \cos x$

$y = x^2 \quad v = \cos x$

$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = -\sin x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$\frac{dy}{dx} = x^2(-\sin x) + \cos x\,(2x)$$

$$\frac{dy}{dx} = -x^2 \sin x + 2x\cos x$$

$$\frac{dy}{dx} = 2x\cos x - x^2 \sin x$$