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Question

For the curves in problem 1 to 5, at the points given, find (a) the equation of the tangent, and (b) the equation of the normal.

1. $y = 2x^2$ at the point $(1, 2)$

2. $y = 3x^2$ at the point $(2, 8)$

3. $y = x^3/2$ at the point $(-1, -1/2)$

4. $y = 4x - x^2$ at the point $(-2, 5)$

5. $y = 4/x$ at the point $(3, 1/3)$

Solution:

1. $y = 2x^2$ at the point $(1, 2)$

$$y = 2x^2$$

$$\frac{dy}{dx} = 4x$$

$$m = \frac{dy}{dx} \Big|_{x=1}$$

$$m = 4(1)$$

$$m = 4$$

a) Equation of the tangent

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 4(x - 1)$$

$$y - 2 = 4x - 4$$

$$y = 4x - 4 + 2$$

$$y = 4x - 2;$$

$$y - 4x + 2 = 0$$

b) Equation of the normal

$$m_2 = \frac{1}{m_1}$$

$$m_2 = -\frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{4}(x - 1)$$

$$y - 2 = -\frac{x}{4} + \frac{1}{4}$$

multiply through by 4

$$4y - 8 = -x + 1$$

$$4y + x - 8 - 1 = 0$$

$$4y + x - 9 = 0$$

2) $y = 3x^2 - 2x$ at the point $(2, 8)$.

$$y = 3x^2 - 2x$$

$$\frac{dy}{dx} = 6x - 2$$

$$m = 6(2) - 2$$

$$m = 10$$

a) Equation of the tangent

$$y - y_1 = m(x - x_1)$$

$$y - 8 = 10(x - 2)$$

$$y - 8 = 10x - 20$$

$$10x - 20 - y + 8 = 0$$

$$y = 10x - 12 = 0$$

b) Equation of the normal

$$m_2 = -\frac{1}{m_1}$$

$$m_2 = -\frac{1}{10}$$

$$y - 8 = -\frac{1}{10}(x - 2)$$

$$y - 8 = -\frac{x}{10} + \frac{1}{5}$$

multiply through by 10

$$10y - 80 = -x + 2$$

$$10y + x - 80 - 2 = 0$$

$$10y + x - 82 = 0$$

3) $y = \frac{x^3}{2}$ at the point $(-1, -\frac{1}{2})$

$$f = \frac{x^3}{2}$$

$$\frac{dy}{dx} = \frac{3x^2}{2}$$

$$m = \frac{dy}{dx} \Big|_{x=1}$$

$$m = \frac{3(-1)^2}{2} = \frac{3}{2}$$

$$m = \frac{3}{2}$$

a) Equation of the tangent:

$$y - y_1 = m(x - x_1)$$

$$y - (-\frac{1}{2}) = \frac{3}{2}(x - 1)$$

$$y + \frac{1}{2} = \frac{3}{2}x + \frac{3}{2}$$

multiply through by 2

$$2y + 1 = 3x + 3$$

$$2y - 3x + 1 - 3 = 0$$

$$2y - 3x - 2 = 0$$

b) Equation of the normal

$$m_2 = -\frac{1}{m_1}$$

$$m_2 = -\frac{1}{\frac{3}{2}}$$

$$m_2 = -\frac{2}{3}$$

$$y + \frac{1}{2} = -\frac{2}{3}(x+1)$$

multiply through by 6.

$$6y + 3 = -4(x+1)$$

$$6y + 3 = -4x - 4$$

$$6y + 4x + 3 + 4 = 0$$

$$6y + 4x + 7 = 0$$

4. $f = 1 + x - x^2$ at the point

$$(-2, -5)$$

$$f = 1 + x - x^2$$

$$\frac{dy}{dx} = 1 - 2x$$

$$m = 1 - (-2) = 3$$

$$m = 1 - (-2) = 3$$

$$m = 3$$

a) Equation of the tangent:

$$y - y_1 = m(x - x_1)$$

$$(y - (-5)) = 3(x - (-2))$$

$$y + 5 = 3(x + 2)$$

$$y + 5 = 3x + 6$$

$$y - 3x + 5 - 6 = 0$$

$$y - 3x - 1 = 0$$

b) Equation of the normal:

$$m_2 = -\frac{1}{m_1}$$

$$m_2 = -\frac{1}{3}$$

$$y + 5 = -\frac{1}{3}(x + 2)$$

Multiply through by 3

$$3y + 15 = 1(x + 2)$$

$$3y + 15 = x + 2$$

$$3y + x + 15 + 0 = 0$$

$$3y + x + 17 = 0$$

5. $y = 1/x$ at the point $(3, 1/3)$

$$y = 1/x$$

$$\frac{dy}{dx} = 1^x - 1/x^2 = -1/x^2$$

$m = \text{dy/dx at } x = 3 = -1/9$

$$m = -1/9$$

$$m = -1/9$$

a) Equation of the tangent

$$y - y_1 = m(x - x_1)$$

$$(6) \quad (y - 1/3) = -1/9(x + 3)$$

Multiply through by 9

$$9y - 3 = -1(x + 3)$$

$$9y - 3 = -x - 3$$

$$9y + x - 3 + 3 = 0$$

$$9y + x - 6 = 0$$

b) Equation of the normal

$$m_2 = -1/m_1$$

$$m_2 = 9$$

$$m_2 = -1/-1/9$$

$$m_2 = 9$$

$y - 1/3 = 9(x - 3)$
 multiply through by 3
 $3y - 1 = 27(x - 3)$

$$3y - 1 = 27x - 81$$

$$3y - 27x - 1 + 81 = 0$$

$$3y - 27x + 80 = 0$$