

1.  $y = 2x^2$  at the point  $(1, 2)$   $\frac{dy}{dx} = 4x$

a.  $m=4$ , Equation of the tangent

$$y - 2 = 4(x - 1) \Rightarrow y - 1 = 4x - 4$$

$$y - 2 - 4x + 4 = 0 \Rightarrow y - 4x + 2 = 0$$

b. Equation of the normal

$$y - 2 = -\frac{1}{4}(x - 1) \Rightarrow y - 2 = \frac{-x + 1}{4}$$

$$\Rightarrow 4(y - 2) = -x + 1$$

$$4y - 8 = -x + 1 \Rightarrow 4y + x - 9 = 0$$

2.  $y = 3x^2 - 2x$  at the point  $(2, 8)$   $\frac{dy}{dx} = 6x - 2$

a. Equation of the tangent  $m = 6 \cdot 2 - 2 = 10$

$$y - 8 = 10(x - 2) \Rightarrow y - 8 = 10x - 20$$

$$y - 8 - 10x + 20 = 0 \Rightarrow y - 10x + 12 = 0$$

b. Equation of the normal  $m_1 m_2 = -1$

$$10m_2 = -1 \Rightarrow m_2 = -\frac{1}{10}$$

$$y - 8 = -\frac{1}{10}(x - 2); 10(y - 8) = -1(x - 2)$$

$$10y - 80 = -x + 2; 10y + x - 80 - 2 = 0$$

$$\Rightarrow 10y + x - 82 = 0$$

3.  $y = \frac{x^3}{2}$  at the point  $(-1, -\frac{1}{2})$

$$\frac{dy}{dx} = \frac{3}{2}x^2 \text{ from } \frac{1}{2}x^3$$

a. Equation of the tangent

$$m = \frac{dy}{dx} = \frac{3}{2}x^2 = \frac{3}{2}$$

$$y - (-\frac{1}{2}) = \frac{3}{2}(x - (-1))$$

$$y + \frac{1}{2} = \frac{3}{2}(x + 1); 2(y + \frac{1}{2}) = 3(x + 1)$$

$$\therefore 2y - 3x - 2 = 0$$

b. Equation of the normal;  $m_1 m_2 = -1$

$$\frac{3}{2}m_2 = -1 \Rightarrow m_2 = -\frac{2}{3}$$

$$y + \frac{1}{2} = -\frac{2}{3}(x + 1); 3(y + \frac{1}{2}) =$$

$$-2(x + 1); 3y + \frac{3}{2} = -2x - 2;$$

$$3y + 2x + \frac{3}{2} + 2 = 0$$

$$\therefore 3y + 2x + \frac{7}{2} = 0$$

4.  $y = 1 + x - x^2$  at the point  $(-2, -5)$

$$\frac{dy}{dx} = 1 - 2x$$

a. Equation of the tangent

$$\frac{dy}{dx} = m = 5$$

$$y - (-5) = 5(x - (-2))$$

$$y + 5 = 5(x + 2); y + 5 = 5x + 10$$

$$y - 5x + 5 - 10 = 0; y - 5x - 5 = 0$$

b. Equation of the normal:

$$m_1 m_2 = -1; 5m_2 = -1; m_2 = -\frac{1}{5}$$

$$y + 5 = -\frac{1}{5}(x + 2); 5(y + 5) = -1(x + 2)$$

$$5y + 25 = -x + 2; 5y + x + 25 - 2 = 0$$

$$\therefore 5y + x + 27 = 0$$

5.  $y = \frac{1}{x}$  at the point  $(3, \frac{1}{3})$

$$\frac{dy}{dx} = -x^{-2} \text{ from } y = x^{-1}$$

a. Equation of the tangent:

$$\frac{dy}{dx} = m = -\frac{1}{9}$$

$$y - \frac{1}{3} = -\frac{1}{9}(x - 3); 9(y - \frac{1}{3}) =$$

$$-1(x - 3); 9y - 3 = -x + 3$$

$$9y + x - 3 - 3 = 0 \Rightarrow 9y + x - 6 = 0$$

b. Equation of the normal:

$$m_1 m_2 = -1; -\frac{1}{9}m_2 = -1; m_2 = 9$$

$$y - \frac{1}{3} = 9(x - 3);$$

$$y - \frac{1}{3} = 9x - 27$$

$$y - 9x - \frac{1}{3} + 27 = 0$$

$$\therefore y - 9x + \frac{80}{3} = 0$$