MAT 104 Thursday 2nd April 2020

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MECHATRONICS ENGINEERING

19/ENG05/1014

1 Find the limit of the function $\frac{x-\cos x}{x}$ $x \to 0$

$\lim_{x \to 0} \left(\frac{x-\cos x}{x} \right)$

Using L'Hopitals rule

$\lim_{x \to 0} \left(\frac{1+\sin x}{1} \right)$

$\lim x \to 0 = \frac{1+\sin 0}{1} = \underline{\underline{1}}$

2 $y = -3\tan 7x\, e^{3x}$, find dy/dx

$y = u\ v\ w$

$\frac{dy}{dx} = uvw\left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}\frac{dw}{dx} \right)$

$u = -3 \quad du/dx = 0$

$v = \tan 7x \quad dv/dx = 7\sec^2 7x$

$w = e^{3x} \quad dw/dx = 3e^{3x}$

$\frac{dy}{dx} = -3\tan 7x\, e^{3x}\left(\frac{1}{-3}\cdot 0 + \frac{1}{\tan 7x} 7\sec^2 7x + \frac{1}{e^{3x}}\cdot 3e^{3x} \right)$

$\frac{dy}{dx} = -3\tan 7x\, e^{3x}\left(\frac{7\sec^2 7x}{\tan 7x} + 3 \right)$

$\frac{dy}{dx} = -3\tan 7x\, e^{3x}\left(7\sec^2 7x \cdot \cot 7x + 3 \right)$

3 $y = \cos 3x$ find dy/dx from the first principle

$y + \Delta y = \cos 3(x + \Delta x)$

$\Delta y = \cos 3(x+\Delta x) - y \qquad y = \cos 3x$

$\Delta y = \cos(3x + 3\Delta x) - \cos 3x$

$\cos A - \cos B = -2\sin\frac{(A+B)}{2} \cdot \sin\left(\frac{A-B}{2}\right)$

$$\Delta y = -2\sin\frac{3x+3\Delta x+3x}{2}\cdot\sin\frac{3x+3\Delta x-3x}{2}$$

$$\Delta y = -2\sin\frac{6x+3\Delta x}{2}\cdot\sin\frac{3\Delta x}{2}$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = -\frac{2\sin\frac{6x+3\Delta x}{2}\cdot\sin\frac{3\Delta x}{2}}{\Delta x}$$

$$\lim_{x\to 0}\frac{\Delta y}{\Delta x} = \frac{3}{2}\left(-2\sin\frac{6x+3\Delta x}{2}\cdot\sin\frac{3\Delta x}{2}\right)\Big/\frac{3}{2}\Delta x$$

$$\lim_{x\to 0}\frac{\Delta y}{\Delta x} = -3\sin\frac{6x+3\Delta x}{2}\cdot\sin\frac{3\Delta x}{2}\Big/\frac{3}{2}\Delta x$$

$$\lim_{x\to 0}\frac{\Delta y}{\Delta x} = -3\sin\frac{6x+3(0)}{2}$$

$$\frac{dy}{dx} = -3\sin\frac{6x}{2}$$

$$dy/dx = -3\sin 3x$$

4 $f(x) = 2x^3 - 7x \quad g(x) = -3x$ find $(f-g)(5)$

$f(x) - g(x) = 2x^3 - 7x - (-3x)$

$f(x) - g(x) = 2x^3 - 7x + 3x$

$f(x) - g(x) = 2x^3 - 4x$

$(f-g)(5) = 2(5)^3 - 4(5)$

$f-g(5) = 250 - 20 = 230$

$= 230$

5 $fog(x) \quad fx = 4x^2 + 2 \quad g(x) = 2x+3$

$f(x) = 4x^2 + 2 \quad x \equiv 2x+3$

$fog(x) = 4(2x+3)^2 + 2$

$fog(x) = 4(4x^2 + 6x + 6x + 9) + 2$

$fog(x) = 4(4x^2 + 12x + 9) + 2$

$fog(x) = 16x^2 + 48x + 36 + 2$

$fog(x) = 16x^2 + 48x + 38$

6 $x^2 + 2xy + y^2 = 1020$

$$2x\frac{dx}{dx} + 2\left[x\frac{dy}{dx} + \frac{y\,dx}{dx}\right] + 2y\frac{dy}{dx} = 0$$

$$2x + 2x\frac{dy}{dx} + 2y\frac{dx}{dx} + 2y\frac{dy}{dx} = 0$$

$$2x + 2y = -2x\frac{dy}{dx} - 2y\frac{dy}{dx}$$

$$2x + 2y = \frac{dy}{dx}\left(-2x - 2y\right)$$

$$\frac{dy}{dx} = \frac{2x + 2y}{-2x - 2y} = \frac{2(x+y)}{-2(x+y)}$$

$$\frac{dy}{dx} = -1$$

7 $y = x^2 \cos x$ find dy/dx

$y = u\,v$

$u = x^2 \quad du/dx = 2x$

$v = \cos x \quad dv/dx = -\sin x$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$\frac{dy}{dx} = \cos x \cdot 2x + x^2 \cdot -\sin x$$

$$\frac{dy}{dx} = 2x\cos x - x^2 \sin x$$