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Differentiate the following

1) $y = \dfrac{[(x+1)^2 (x-2)^{1/2}]}{[(2x-1)(x+3)^{3/2}]}$ (2) $y = \dfrac{[3e^k \sin 2k]}{k^{5/2}}$

SOLUTION

1) $y = \dfrac{[(x+1)^2 (x-2)^{1/2}]}{[(2x-1)(x+3)^{3/2}]}$

$\ln y = [\ln (x+1)^2 + \ln (x-2)^{1/2}] - [\ln (2x-1) + \ln (x+3)^{3/2}]$

$\cancel{\frac{1}{y} \cdot \frac{dy}{dx}} \ln y = [2 \ln (x+1) + \tfrac{1}{2} \ln (x-2)] - [\ln (2x-1) + \tfrac{3}{2} \ln (x+3)]$

$\frac{1}{y} \cdot \frac{dy}{dx} = \left[2 \cdot \frac{1}{x+1} \cdot 1 + \frac{1}{2} \cdot \frac{1}{x-2} \cdot 1\right] - \left[\frac{1}{2x-1} \cdot 2 + \frac{3}{2} \cdot \frac{1}{x+3} \cdot 1\right]$

$\frac{1}{y} \cdot \frac{dy}{dx} = \left[\left(\frac{2}{x+1} + \frac{1}{2x-4}\right) - \left(\frac{2}{2x-1} + \frac{3}{2x+6}\right)\right]$

$\frac{1}{y} \cdot \frac{dy}{dx} = \left[\frac{2}{x+1} + \frac{1}{2x-4} - \frac{2}{2x-1} - \frac{3}{2x+6}\right]$

$\frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{1}{2x-4} - \frac{2}{2x-1} - \frac{3}{2x+6}\right]$

$\frac{dy}{dx} = \frac{[(x+1)^2 (x-2)^{1/2}]}{[(2x-1)(x+3)^{3/2}]} \cdot \left[\frac{2}{x+1} + \frac{1}{2x-4} - \frac{2}{2x-1} - \frac{3}{2x+6}\right]$

2) $y = \dfrac{[3e^k \sin 2k]}{k^{5/2}}$

$\ln y = \ln 3e^k + \ln \sin 2\cancel{k} - \ln k^{5/2}$

$\ln y = \ln 3e^k + \ln \sin 2k - \tfrac{5}{2} \ln k$

$\frac{1}{y} \cdot \frac{dy}{dk} = \frac{1}{3e^k} \cdot 3e^k + \frac{1}{\sin 2k} \cdot 2\cos 2k - \frac{5}{2} \cdot \frac{1}{k} \cdot 1$

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$$\frac{1}{y} \cdot \frac{dy}{dk} = 1 + 2\cot 2k - \frac{5}{2k}$$

$$\frac{dy}{dk} = y\left[1 + 2\cot 2k - \frac{5}{2k}\right]$$

$$\frac{dy}{dk} = \left[\frac{3e^{k}\sin 2k}{k^{5/2}}\right]\left[1 + 2\cot 2k - \frac{5}{2k}\right]$$

Integrate the following with respect to the variable.

1) $4\sec^2(3m+1)$ 2) $2t(3t^2-1)^{1/2}$ 3) $2x/(4x^2-1)^{1/2}$

1) $4\sec^2(3m+1)\,dm$

$$4\int \sec^2(3m+1)\,dm$$

let $u = 3m+1$

$du/dm = 3$

$dm = du/3$

$$\therefore 4\int \sec^2 u \cdot du/3$$

$$= \frac{4}{3}\tan u + C$$

where $u = 3m+1$

$$= \frac{4}{3}\tan(3m+1) + C$$

2) $2t(3t^2-1)^{1/2}\,dt$

let $u = 3t^2-1$

$du/dt = 6t$

$$dt = \frac{du}{6t}$$

$$\int 2t\,u^{1/2} \cdot \frac{du}{6t}$$

$$\int \frac{\cancel{2t}\,u^{1/2}}{3\,\cancel{6t}} \cdot du = \frac{1}{3}\int u^{1/2} \cdot du$$

$$= \frac{1}{3}\left[\frac{u^{1/2+1}}{1/2+1}\right] + c$$

$$= \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]$$

$$= \frac{1}{3}\left[\frac{2u^{3/2}}{3}\right]$$

$$= \frac{2u^{3/2}}{9} + C$$

where $u = 3t^2 - 1$

$$= \frac{2(3t^2-1)^{3/2}}{9} + C$$

3) $\displaystyle\int \frac{2x}{(4x^2-1)^{1/2}}\, dx$

Let $u = 4x^2 - 1$

$du/dx = 8x$

$dx = \frac{du}{8x}$

$$\int \frac{\cancel{2x}}{u^{1/2}} \cdot \frac{du}{\cancel{8x}\,4}$$

$$\int u^{-1/2} \cdot \frac{du}{4}$$

$$= \frac{1}{4}\int u^{-1/2} \cdot du$$

$$= \frac{1}{4}\left[\frac{u^{-1/2+1}}{-1/2+1}\right] + C$$

$$= \frac{1}{4}\left[\frac{u^{1/2}}{1/2}\right] + C$$

$$= \frac{1}{2}u^{1/2} + C$$

where $u = 4x^2 - 1$

$$= \frac{(4x^2-1)^{1/2}}{2} + C$$