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19/ENG05/019
MAT 104
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1 $\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\}$

Solution

$\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\}$

By direct substitution, we have $\frac{0}{0}$: Using L'hopital's rule

$\lim_{x \to} \left\{ \frac{x - \cos x}{x} \right\} = \lim_{x \to 0} \left\{ \frac{1 - (-\sin x)}{1} \right\}$

$= \lim_{x \to 0} \left\{ \frac{1 + \sin x}{1} \right\} = \frac{1 + \sin 0}{1} = \frac{1+0}{1}$

$= 1$

2 $y = -3 \tan 7x \, e^{3x}$

Solution

$y = -3 \tan 7x \, e^{3x}$

$u = -3 \quad v = \tan 7x \quad w = e^{3x}$

$du/dx = 0 \quad dv/dx = 7\sec^2 7x \quad dw/dx = 3e^{3x}$

$$dy/dx = y\left[\frac{1}{u} \times \frac{du}{dx} + \frac{1}{v} \times \frac{dv}{dx} + \frac{1}{w} \times \frac{dw}{dx}\right]$$

$$= y\left[\frac{1}{-3} \times 0 + \frac{1}{\tan 7x} \times 7\sec^2 7x + \frac{1}{e^{3x}} \times 3e^{3x}\right]$$

$$= y\left[0 + \frac{7\sec^2 7x}{\tan 7x} + 3\right]$$

$$dy/dx = -3\tan 7x \, e^{3x}\left[\frac{7\sec^2 7x}{\tan 7x} + 3\right]$$

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3. $y = \cos 3x$

Solution

$y = \cos 3x$

$y + \Delta y = \cos 3(x + \Delta x)$

$y + \Delta y = \cos 3x + 3\Delta x$

$\Delta y = \cos 3x + 3\Delta x - \cos 3x \quad \cdots ①$

Recall $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \quad \cdots ②$

Comparing ① and ②

$A = 3x + 3\Delta x \qquad B = 3x$

$\frac{A+B}{2} = \frac{3x + 3\Delta x + 3x}{2} = \frac{6x + 3\Delta x}{2} = 3x + \frac{3\Delta x}{2}$

$\frac{A-B}{2} = \frac{3x + 3\Delta x - 3x}{2} = \frac{3\Delta x}{2}$

Hence $\Delta y = -2 \sin\left(3x + \frac{3\Delta x}{2}\right) \sin\left(\frac{3\Delta x}{2}\right)$

divide through by Δx

$\Delta y/\Delta x = \dfrac{-2 \sin\left(3x + \frac{3\Delta x}{2}\right) \sin\left(\frac{3\Delta x}{2}\right)}{\Delta x}$

Multiply both numerator and denominator

~~Divide~~ through by $3/2$

$\dfrac{-2 \sin\left(3x + \frac{3\Delta x}{2}\right) \sin\left(\frac{3\Delta x}{2}\right) \times 3/2}{\Delta x \times 3/2}$

$\Delta y/\Delta x = \dfrac{-3 \sin\left(3x + \frac{3\Delta x}{2}\right) \sin\left(\frac{3\Delta x}{2}\right)}{3\Delta x/2}$

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3 cont'd

$\lim_{\Delta x \to 0} \Delta y/\Delta x = -3\sin 3x$

$\lim_{x \to 0} \frac{\sin \frac{3\Delta x}{2}}{\frac{3\Delta x}{2}} = 1$

~~Hence~~ $\therefore -3\sin 3x \times 1$

Hence $dy/dx = -3\sin 3x$

4 $f(x) = 2x^3 - 7x$

$g(x) = -3x$

$f(x) - g(x) = 2x^3 - 7x - (-3x)$

$= 2x^3 - 7x + 3x$

$= 2x^3 - 4x$

$(f - g)(5) = (2x^3 - 4x) \times 5$

$= 10x^3 - 20x$

dividing through by 10

$x^3 - 2x$

5 $f \circ g(x)$

$f(x) = 4x^2 + 2$

$g(x) = 2x + 3$

$fog(x) = 4(2x+3)^2 + 2$

$= 4(4x^2 + 12x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$= 16x^2 + 48x + 38$

$fog(x) = 16x^2 + 48x + 38$

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6 $x^2 + 2xy + y^2 = 1020$

Solution

$x^2 + 2xy + y^2 = 1020$

$2x\frac{dx}{dx} + 2x\,dy/dx + 2y\frac{dx}{dx} + 2y\,dy/dx = 0$

$2x + 2x\,dy/dx + 2y + 2y\,dy/dx = 0$

$2x + 2y + dy/dx(2x + 2y) = 0$

$dy/dx(2x + 2y) = -2x - 2y$

$dy/dx = \frac{-2x - 2y}{2x + 2y}$

7 $y = x^2 \cos x$

Solution

Put $u = x^2$, $v = \cos x$

$du/dx = 2x$, $dv/dx = -\sin x$

Recall $u \cdot dv/dx + v \cdot du/dx$

$= x^2 \cdot -\sin x + \cos x \cdot 2x$

$= -x^2 \sin x + 2x \cos x$

$= 2x\cos x - x^2 \sin x$